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Estimating the hydrologic induced signal in geodetic measurements with predictive filtering methods

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Abstract. The high precision achieved with continuous geodetic instruments makes it necessary to take ambient factors into account. Among these, one of the most disturbing is the hydrologic agent. After giving a characterization of the induced signals in the specific case of subsurface tilt and extensometric measurements, the techniques of predictive filtering are shown to solve the problem of modeling the induced signals. The results obtained here may be applied also to other continuous geodetic and gravity measurements.

Introduction

Increasing the accuracy in geodetic measurements or in continuous gravity measurements makes a precise knowledge of the influence of ambient factors ever more important. Among these, the temperature, atmospheric pressure and the hydrologic agents are to be considered. In the present study we focus on the hydrologic agents, which divide into precipitation and water table variation. We consider the influence of rainfall only, the water table variation being generally not independent from precipitation. The importance of the hydrologic agents in geodetic measurements has been recognized for several decades. The effect is spread out over a large band of frequencies, ranging from short term (days) to long term (years) variations (*Edge et al.*, 1981; *Peters and Beaumont*, 1981; *Wolfe et al.*, 1981; *Kasahara et al.*, 1983; *Yamauchi*, 1987; *Tanaka et al.*, 1989; *Dal Moro and Zadro*, 1998; *Weise et al.*, 1999).

The physical nature of the induced deformation is not fully understood and different physical models have been proposed. Generally speaking, pore pressure is assumed responsible for the observed phenomena. In particular *Evans and Wyatt* [1984] assume changes in the aperture of subsurface hydraulically conductive fractures accompanying pore pressure changes. The model of deformation of the rock matrix induced by pore pressure gradients leading to groundwater flow in the pore space has been found to be adequate to explain the observations of tilt and strainmeters. *Kümpel* [1989] has examined this model and made pumping experiments in order to test the theory. The approach of physically modeling the induced signals is very difficult, as the finite element model requires detailed knowledge of the geological and hydrological structure at the observation site. The works of *Kümpel* [1989] and *Weise et al.* [1999] showed that in the examined cases the order of magnitude of the observed induced signal is in agreement with the one predicted by the model.

Statistical models have had greater popularity, and different techniques have been proposed to tackle the problem. *Langbein et al.* [1990] convolve the rainfall with an exponential function of a certain time constant and calculate the cumulative function of rainfall, which should model the observations. *Wolfe et al.* [1981] apply models of known hydrologic mechanisms, as infiltration and outflow of water from the ground, in order to find empirical models for the rain-strain relationship. *Yamauchi* [1987] uses an alternative method for simulating groundwater flow, which considers concatenated tanks fed in one another. The outflow from the last tank is used to simulate the strain variation. This last method gave promising results, but is challenging to apply, as the nonlinear equations require parameter adaptation by trial and error. In the following we show that the techniques of predictive filtering give good results for estimating the hydrologic induced deformation.

The techniques are applied to extensometric and tilt measurements made at the geodetic network of NE-Italy, installed in 1977 [*Zadro*, 1978]. The network is set in a seismically active area which was struck in the last 25 years by two destructive events of M=6.4 in 1976 and M=5.6 in 1998. The resolution and underground housing of the instrumentation is of sufficient quality to expect the observation of deformation connected to the pre-, co- and postseismic phases of a local seismic event of greater magnitude. The test site Gemona houses two Zöllner type tiltmeters. The Villanova station is equipped, in addition to other instruments, with 3 horizontal Cambridge type Invar strainmeters (length between 10 and 13 m) installed at 60-m depth. Details on the network are found in *Braitenberg* [1998]. The pluviometer (Vedronza) is installed at a distance of 9 km and 2km to the stations Gemona and Villanova, respectively. The rainfall data are furnished by the Italian Government Service (Magistrato delle Acque).

Modeling

We aim at modeling the hydrologic induced signals in geodetic measurements by considering the direct influence of rainfall on the data. The use of the proposed technique is restricted to the short period hydrologic effects, with variations between a day and a month. This range of frequencies is the most evident and most disturbing of the hydrologic induced signals in geodetic measurements. The induced signals are tied to the onset of rainfall, which generally has the effect of an impulse-like deformation with exponential recovery [*Langbein et al.*, 1990]. Longer period variations, up to a year and more, as observed by *Kasahara et al.* [1983], are due to fluctuations in the mean precipitation rate and must be studied with alternative methods, also taking into account the water table. The method here proposed makes use of the Autoregressive Moving Average (ARMA) time series model to approximate the series. The approach of using an ARMA model has been successfully used by *Matsumoto* [1992] to estimate the rainfall effect on the groundwater level. The model is represented by the filter linear

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difference equation

$$x(n) = -\sum_{k=1}^{p} a(k)x(n-k) + \sum_{k=0}^{q} b(k)u(n-k) \qquad (1)$$

in which $x(n)$ is the output sequence of a causal filter that models the observed data and $u(n)$ is an input driving sequence [*Marple*, 1987; p.174]. The $a(k)$ parameters form the autoregressive portion, the $b(k)$ parameters the moving average portion of the model. The parameters p and q indicate the AR and MA order of the model, respectively. It can be shown that an ARMA model of a finite number of parameters can be represented by an AR model of generally infinite order [*Marple*, 1987]. In our specific application we use the AR models, as they have the advantage that the AR parameters can be obtained as solutions to linear equations, and many efficient algorithms exist. MA and ARMA models in the contrary require the solution of nonlinear equations. The use of AR models for an ARMA or MA process is possible, as long as the model is allowed to have a sufficiently large order. A pure AR model is defined as

$$x(n) = -\sum_{k=1}^{p} a(k)x(n-k) + u(n) \qquad (2)$$

The estimation of AR parameters can be done with block-estimation methods, in which the entire set of data is considered, or with adaptive algorithms. With adaptive algorithms the AR parameters estimates are updated in time. These techniques are useful in the case of slowly varying characteristics.

The algorithms for the calculation of the AR parameters require the data sequence and the model order as input. In practice it is necessary to postulate several model orders, and choose an error criterion that indicates which model order to choose. Criteria such as the final prediction error [*Akaike*, 1969], the Akaike information criterion (AIC) [*Akaike*, 1974] and the criterion of autoregressive function (CAT) of *Parzen* [1974] have been shown to work well with synthetic signals. In practice though, with the presence of noise and the data series being not purely AR, the above criteria have been found to give erroneous results [*Marple*, 1987, ff. 229] usually underestimating the model order.

In the application to the hydrologic signals we introduce a specific criterion, which considers the forward prediction error (fpe) of the AR model and the observed rainfall. The AR model should represent the hydrologic induced deformation, the driving sequence being the rainfall multiplied by an appropriate scaling factor. The driving sequence is equal to the fpe, which given the AR parameters is obtained from

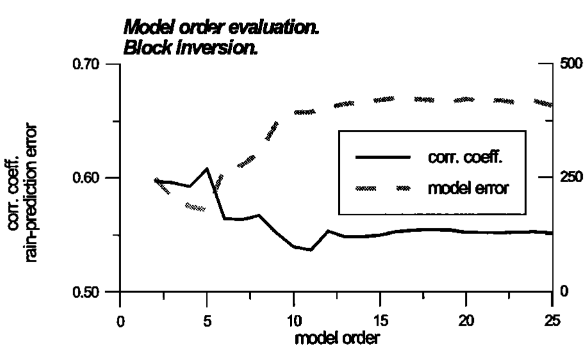

Figure 1. Evaluation of the most appropriate model order of the block inversion: maximizing the correlation coefficient of rainfall and the theoretical driving sequence obtained from the AR model for different orders of the AR model, or minimizing the root mean square of the difference of the observed and predicted rain-induced tilt for different model orders. For the tilt records the quantities are extreme for model order equal to 5.

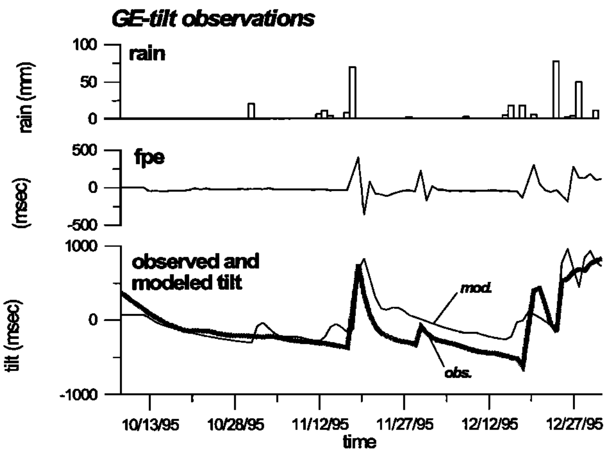

Figure 2. Modeling of the hydrologically-induced tilt signal at GE-station. Observed tilt records have been reduced to daily sampling and long period signals have been removed. Rainfall, forward prediction error of modeling (fpe) and modeled (light trace) and observed (heavy trace) hydrologically-induced deformation of tilt are shown.

$$e_p(n) = -\sum_{k=1}^{p} a(k)x(n-k) + x(n) \qquad (3).$$

That order p is chosen, for which the correlation coefficient of the fpe ($e_p(n)$) of the model of order p and the rainfall ($r(n)$) is maximal:

$$\gamma_p = \frac{\sum_{n=1}^{N}\left(e_p(n) - \bar{e}_p\right)\left(r(n) - \bar{r}\right)}{\sqrt{\sum_{n=1}^{N}\left(e_p(n) - \bar{e}_p\right)^2 \sum_{n=1}^{N}\left(r(n) - \bar{r}\right)^2}} \qquad (4)$$

Alternatively we may define the model error of the induced signal with

$$\Delta_p(n) = x(n) - \left[S_p(r(n) - \bar{r}) - \sum_{k=1}^{p} a_p(k)x(n-k) \right] \qquad (5),$$

in which S_p is a scaling factor. The scaling factor (S_p) is equal to the ratio of the mean square root amplitude of the fpe ($e_p(n)$) and the rainfall ($r(n)$). That order p is chosen for which the mean square root amplitude of the model error ($\Delta_p(n)$) is minimized.

Both the block estimation method and the adaptive algorithm have been tested on the deformational records of the Friuli tilt-strainmeter network. The block estimation method is preferable in the case a short data series is to be analyzed, as the adaptive algorithm is recursive and necessitates a part of the sequence until it stabilizes to the correct AR-parameters. A well-studied implementation is the Harmonic (Burg) algorithm, also called maximum entropy algorithm, which was introduced in 1967 [see e.g. *Marple*, 1987].

As an example of the block method analysis, we show the application to 2.5 months of tilt recordings of the GE (Gemona) station. The original hourly sampled data have been reduced to a daily sampling rate. Secular drift and the yearly thermoelastic cycle have been modeled as a polynomial of order 4 and a

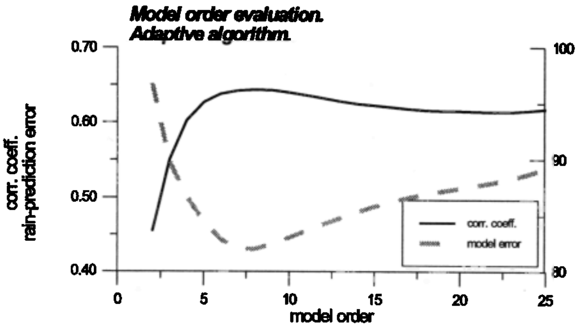

Figure 3. Evaluation of the most appropriate model order of the adaptive filter: maximizing the correlation coefficient of rainfall and the theoretical driving sequence obtained for different orders of the AR model, or minimizing the root mean square of the difference of the observed and predicted rain induced strain for different model orders. For the extensometric record the quantities are extreme for model order equal to 7.

sinusoid of 1 year period which have been estimated by least mean squares evaluation and subtracted. Residual slow deformations have been taken off by high pass filtering with a Hamming filter having a cutoff period of 30 days [*Hamming*, 1962, pp. 297-299]. We take advantage of the fact that the rain-induced tilt has a preferential direction, which for this particular station and year has been found to be N86E. For the modeling we consider the tilt component aligned along this preferential direction. The preferential direction has no obvious relation to structure, which due to the superposition of different systems has no clear directionality. Different AR- model orders have been tested, as shown in Figure 1. The correlation coefficient between the inducing rainfall and the fpe is maximum for the order 5. The same order minimizes the root mean square (rms) amplitude of the difference between observed and predicted tilt (model error). The resulting series are shown in Figure 2, where the light trace is the modeled hydrologic induced effect, the heavy trace the observed data. Also shown in the figure are the rainfall and the fpe. The scaling factor S resulted to be S=11 msec/mm (53 nrad/mm) rainfall.

The adaptive algorithms have the advantage that the AR-parameters are modified in time, but the analysis requires a longer data series. A robust adaptive method is the gradient least mean squares (LMS) algorithm. A discussion of the properties of the gradient LMS algorithm is found for example in *Marple* [1987] and *Haykin* [1996]. The adaptive algorithm has been applied to one component of the extensometric records of station Villanova over a period of six months. The instrument is an invar wire Cambridge type extensometer of 13 m length, installed at an azimuth of 68° [*Zadro*, 1992; *Braitenberg*, 1998]. As before, the secular and annual components have been estimated by least mean square evaluation of a polynomial and a sinusoid. Residual long period signals have been taken off by high pass filtering with cut off period of 30 d. The time window chosen is particularly interesting, as on April 12, 1998 the Slovenian (Bovec) M=5.6 event occurred at 30-km epicentral distance from the station. The event was recorded as a step like deformation of -234 nstrain, which could be modeled by a dislocation [*Braitenberg*, 1998]. For the purpose of the present analysis, the strain step has been taken out.

With the adaptive gradient LMS algorithm the AR-parameters are iteratively corrected according to

$$a_k(n+1) = a_k(n) - \nu \frac{\partial}{\partial a_k}(e_p^2(n)) \qquad k = 1, p \qquad (6)$$

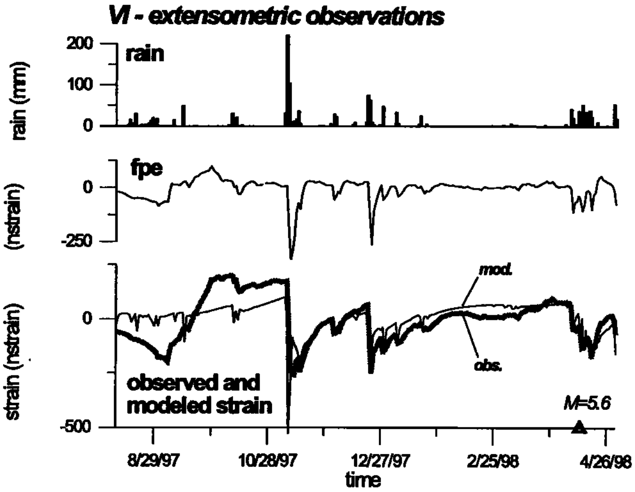

Figure 4. Extensometric records at station Villanova for the period Nov. 1, 1997 – Apr. 30, 1998. The onset time of the Slovenian M = 5.7 event is April 12, 1998 (black triangle). The coseismic step has been taken out. Rainfall, forward prediction error of modeling (fpe) and modeled (light trace) and observed (heavy trace) hydrologically-induced deformation of tilt are shown.

with $e_p(n)$ the fpe at the step n, and ν a convergence factor, which adjusts the amount of correction made at each step. In order to ensure stability of the algorithm, the convergence factor must be chosen as $\nu < 1/pw$, with w the mean square amplitude of the fpe [*Marple*, 1987]. The choice of the parameter ν is governed by the tradeoff between the adaptation rate of the algorithm and the stability with respect to noise in the data. For the above extensometric measurements we have chosen a value of ν=10^{-7} 1/nstrain^2. The model order was chosen with the criterion of maximizing the correlation factor of rainfall and linear forward prediction error and minimizing the mean square error of the predicted strain signal (model error). The criteria are both fulfilled for an order of 7 (Figure 3). The scaling factor amounts to S=3 nstrain/mm. The observed extensometric record and the modeled hydrologic induced deformation are graphed in Figure 4. As in Figure 2, the rainfall and the fpe are graphed as well. The strong resemblance between the rainfall and the fpe is evident, at least for the second two thirds of the displayed record. This is an evidence for the fact that the rainfall has an important role as driving sequence to the predictive model describing the

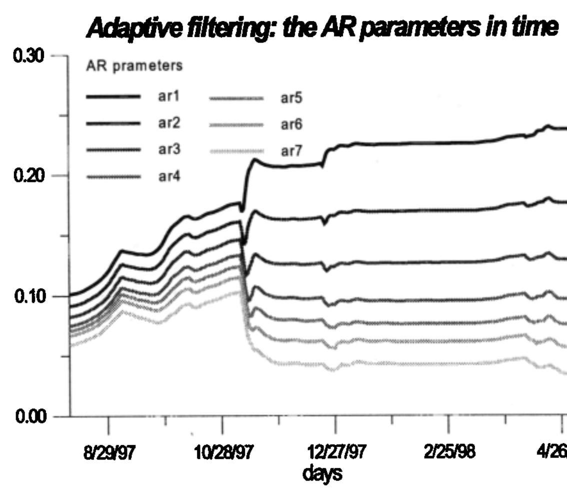

Figure 5. The parameters of the AR model of the extensometric records at station Villanova in time, as they result from the adaptive AR method of order 7.

observed data. For the first third of the record a source other than rainfall must be at the origin of the observed signals. The modeled rain-induced signal confirms the above statement, as it reproduces the observed signal very well in the last two thirds of the sequence. The method allows the strong deformation signals observed before and after the seismic event to be attributed unequivocally to the presence of rainfall, and not to a possible pre/post seismic deformation. The time variation of the parameters making up the AR-model is shown in Figure 5. The sharp variation in the AR-parameters, which follows the first strong rainfall, may reflect the rainfall-dependence of the physical properties such as water run-off and the coefficient of infiltration.

Discussion and conclusion

The observed deformations are a combination of the signals of tectonic origin and those caused by ambient factors, among which the hydrologic agent is the most important. The predictive filtering method is a means to represent the observed data by the output of a linear system, which is fed by a driving random sequence. In the general case the system is defined as being autoregressive moving average, defined by a series of parameters. The parameters characterizing the system can be obtained from the observations, by application of suitable algorithms, as those described above. Once the parameters defining the system have been determined, there are two approaches that can be used to test whether and to what extent the observed deformation is rain-induced.

On one hand, with the observations and the parameters describing the system, the predictive filtering method allows the expected driving sequence to be calculated (equal to fpe, see eq. 3). The correlation coefficient between rainfall and the expected driving sequence (fpe) is a means to estimate to which extent the observations are rain-induced. In the two examples shown above, the correlation factor is 0.61 for the tilt and 0.64 for the extensometric observations. These values are to be compared with the correlation factors of rainfall and tilt (0.1) or rainfall and the extensometric record (0.04) for the same period, which are misleading and would bring to the false conclusion that the observations were independent of rainfall. Conversely, the rainfall can be used as the driving sequence of the system in order to model the rain induced signal. For the tilt and strain measurements this method has allowed successful modeling of the rain-induced deformation. Nonetheless some mismatch of the modeled signal occurs as e.g. in Figure 2, where small rainfall did not result in an induced deformation, contrarily to what predicted from the model. In a further study it should be examined whether preprocessing of the rainfall improves the modeled deformation. Physically, the preprocessing could take account of effects as the maximum infiltration rate of the soil, or a threshold amount of rain before any effect to occur.

The increase in the quality of other geodetic measurements, such as the superconducting gravimeter and continuous GPS observations, has revealed that these are also affected by the hydrologic agents. If not recognized or corrected for, the induced signals are liable to be confused or erroneously interpreted as of tectonic origin. The procedure shown in the present paper to verify and model hydrologically induced signals in extensometric and tilt measurements can be applied in the same manner to other geodetic measurements as well, such as continuous GPS and gravity measurements.

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