

# Constraining the Continental Crust Radioactive Heat Production with Satellite-derived Gravity Models

Alberto Pastorutti\* and Carla Braitenberg  
Dept. of Mathematics and Geosciences, Univ. of Trieste, Italy  
NGTS 36, Trieste 14-16 November 2017

3.3

## 1 BACKGROUND

The resolution of satellite-derived global gravity models (GGMs) is adequate to resolve the mass distribution in the continental crust, the strong density contrast at the crust-mantle boundary (CMB), and the undulations of the lithosphere-asthenosphere boundary (LAB). These aspects suggest that GGMs can be promising tools in modelling the deep thermal state of the lithosphere, the heat transfer regimes involved and the heat flow through the Earth surface. The directly measurable near-surface temperature field is largely influenced by ongoing geodynamics and near-surface processes, both of which have shorter characteristic timescales than the one needed by purely conductive thermal diffusion to reach steady-state in the lithosphere. Heat flow measurements are also costly, their distribution is often biased towards areas of increased interest (e.g. those with high fluxes, exploited for high-enthalpy geothermal energy) and public access to data is an issue. Collecting and harmonising the published datasets to a common standard is an effort spanning multiple decades [1].

Gravity and geoid anomalies have already been integrated in multi-observable modelling strategies, and show a satisfactory resolving power for investigating the nature of lithospheric inhomogeneities [2]. Still, satellite-derived gravity data alone – which has an unmatched global sampling regularity – can already provide estimates independently from other geophysical data, before integration. A relationship between the lithospheric mass distribution (inverted from density contrasts) and models of its thermal state must rely on laws connecting density and thermal parameters (i.e. radioactive heat production, thermal conductivity, boundary conditions), and a set of hypotheses on the heat transport mechanisms involved. A key factor is the radioactive heat production (RHP) occurring in the crystalline continental crust, which exhibits a 50-fold increase against sub-continental mantle content in U, Th, K [3] and is a major component of the surface heat flow even when superimposed with concurring near-surface disturbances [4].

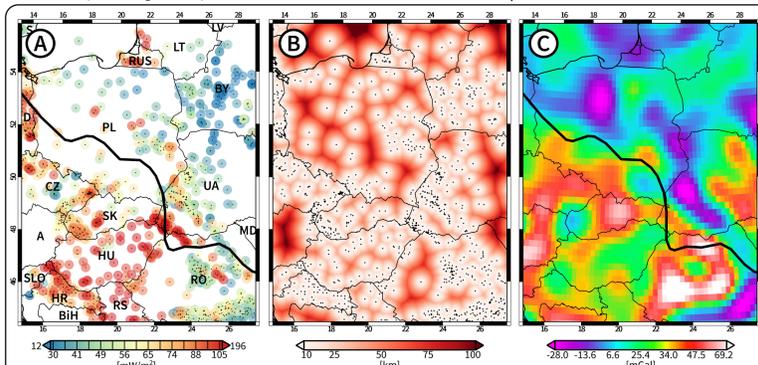


Figure 1: State of the available data: the difference between thermal measurements and satellite gravity models. An example in Central Europe, across the Trans-European Suture Zone (shown as the 150 km lithosphere thickness contour, from [11]), an area with a relatively dense heat flow sample distribution. Map A: heat flow measurements, as publicly available in [12]. Map B: map of the distance from the nearest measurement, in km. Map C: Free Air gravity anomaly at 8 km over GRS80, calculated from the GOCE-derived GO CONS GCF 2 TIM R5 global gravity model (oversampled to 0.25 deg/px).

Estimating the distribution of these elements occurring throughout the continental crust is not a trivial task, since direct and indirect observations (outcrops, xenoliths, and tomographies [5, 6, 7]) indicate that any simple relationship between crustal thickness and heat production [8] is complicated by the large intra-crustal compositional variability. For such reasons stochastic approaches are commonly employed, either exact solutions [9] or random modelling [10], and the results are commonly described with their probability density function. Apart from parametric uncertainty, the entity and predictability of the relationship between crustal thickness and total heat production is difficult to evaluate on itself, due to aforementioned superposition of effects in the observed surface heat flow.

While 1D vertical heat transport is expected as a first order mechanism (the steepest gradient in the lithosphere is always the one from LAB to surface), horizontal conduction arises from horizontal discontinuities in radiogenic heat production, in thermal conductivity and in variation of the boundary conditions (i.e. the thermal thickness of the lithosphere). With the aim of obtaining reliable results from joint heat flow - gravity field modelling, such as the isolation of the crustal component from surface measurements, or the inversion of structures (e.g. basins) that can rearrange the regional thermal regime, we show how horizontal heat transport complicates the interpretation of surface heat flow, even before taking into account the complex variability of thermal parameters. An aspect of particular interest is how the relationship between crustal thickness and bulk heat production is affected by this - hindering the straightforward application of gravimetric Mohos to obtain thermal insights. This required the setup of a simple and efficient solver for the heat equation on 3D volumes, adequate for the problems under enquiry and further developments.

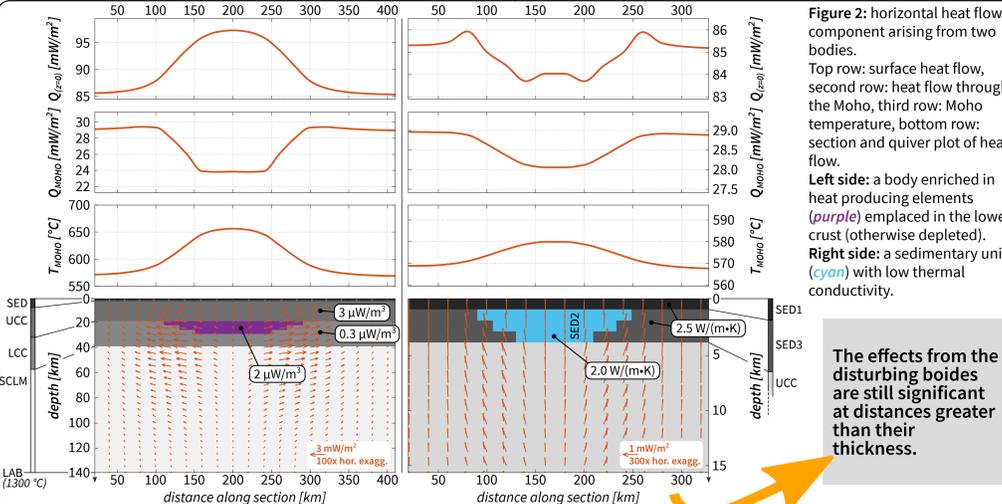


Figure 2: horizontal heat flow component arising from two bodies. Top row: surface heat flow, second row: heat flow through the Moho, third row: Moho temperature, bottom row: section and quiver plot of heat flow. Left side: a body enriched in heat producing elements (purple) emplaced in the lower crust (otherwise depleted). Right side: a sedimentary unit (cyan) with low thermal conductivity.

The effects from the disturbing bodies are still significant at distances greater than their thickness.

## 2 METHODS

We developed a 3D finite-difference forward modelling solver, on rectilinear domains, with non-homogeneous heat production and conductivity. It solves the steady state diffusion equation in the form:

$$\nabla(k(x)\nabla T(x)) = -A(x)$$

where  $k$  is the thermal conductivity,  $A$  is the heat production per unit of volume, and  $x$  is the position vector. The adopted finite difference scheme is a geometrically simple configuration, which imposes some limitations on the shape of the modelled bodies: we are employing a block discretisation. These limitations are outweighed by the lean code and the light implementation (less than 5% of runtime is spent on building the coefficient matrix). The smoothing effect carried out by heat diffusion is such that the effect of sharp steps at depth is negligible at the surface. Another advantage is that the rectilinear discretisation is coherent with the commonly used prism discretisation adopted in gravity modelling, a much needed aspect when planning to carry out joint modelling.

The input consists of  $k$  and  $A$ , as volumes of nodes of the rectilinear grid. Using a dedicated function, models can be defined layer-wise and translated into volumes, a flexible approach to intuitively include both synthetic experiments and real data. The layer definition can include the depth dependence of  $k$  and  $A$  (which is computed when the volume is filled) or the temperature dependence of  $k$  (which can be taken into account iteratively, starting from an initial temperature guess).

The coefficient matrix is inverted with a direct solver based on the Cholesky decomposition (CHOLMOD). When memory becomes a constraint, the script can revert to the generalised minimal residual method, an iterative solver. The whole framework is implemented in Matlab. In regard to the gravity part, forward modelling is carried out with a prism based algorithm, while the inverse modelling relies on an iterative constrained inversion routine [12].

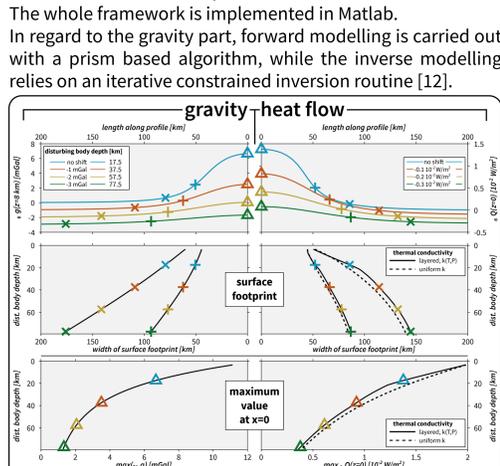


Figure 3: A disturbing body (80x2 km sill-like, +0.2 kg/m³, +2 μW/m³) at varying depth. Left: g anomaly at 8 km height. Right: surface heat flow. We show the width of the surface footprint (0.5 times the value at x=0) and the maximum anomaly.

## 3 STRATEGY

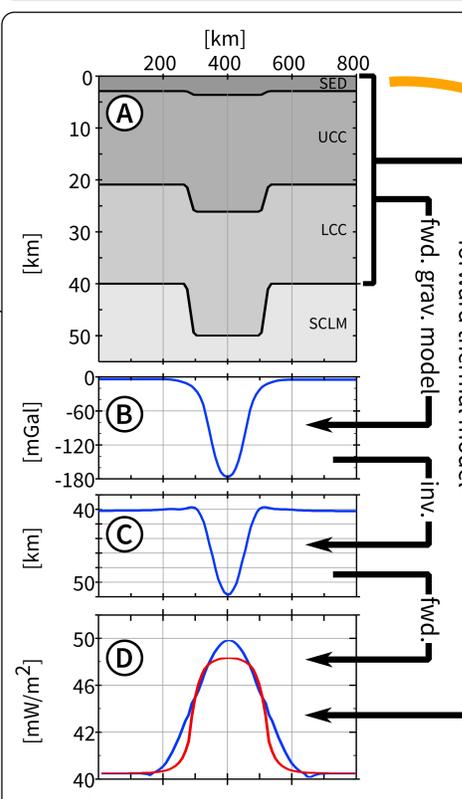


Figure 4: A forward-invert-restore test under ideal conditions. A 10 km crustal thickening of a linearly scaling crust. The lithosphere thickness is constant and the undisturbed Moho depth is known (reference at 0 mGal). The forward gravity (B) is computed at an altitude of 8 km and low-pass filtered at 70 km, to simulate its retrieval from a satellite-derived gravity model up to N=280. (C) shows the apparent Moho obtained from 2-layer inversion, from which a crustal model is retrieved. The same scaling relationship occurring in (A) is used. In (D) both the true surface heat flow and the one retrieved from the gravity inversion are shown.

A straightforward strategy: adopt a reference crust and scale it to the crust thickness obtained through gravity inversion. We test this in figure 4, under ideal conditions.

The difference between the inverted-restore surface heat flow (blue) from the true one (red) is all due to the upward continuation of gravity (loss of higher frequencies) and the re-mapping of a multilayer structure to a 2-layer crust-mantle model.

## 4 REVISITING THE LINEAR RELATIONSHIP

In the example of figure 4 the depth-wise distribution of heat production in the reference crust was known. We also knew that it perfectly scaled with crustal thickness (an ideal condition: e.g. shortening and thrusting of two identical crusts).

What if, having an ideal coverage of surface heat flow measurements, free from any near-surface noise, we try to construct a linear relationship between crustal thickness and surface heat flow?

This equals to the traditional linear model [see 8]

$$Q_0 = A \cdot D + Q_m$$

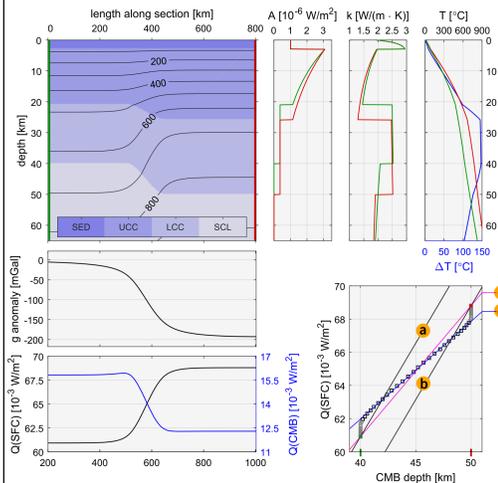


Figure 5: Top left: the model section. Top right: the depth-wise distribution of radioactive heat production (A), thermal conductivity (k) and temperature (T). The green and red lines refer to two crustal columns, far from the lateral transition (see markers), the blue line represents the temperature difference between the two. Q(SFC): surface heat flow, Q(CMB): heat flow at the Moho. Bottom right: crustal thickness and surface heat flow. Linear fits:

	slope [μW/m²]	intercept [mW/m²]
a) true condition for the 40 km crust	1.13	15.82
b) true condition for the 50 km crust	1.13	12.29
c) result of fitting with data at the two markers	0.79	29.39
d) result of fitting in the sloped zone	0.59	38.42

## 5 CONCLUSIONS AND OUTLOOK

- By interpreting these fits as slope = average heat production (A), intercept = basal heat flow (Q<sub>m</sub>) we get:
  - a, b: increase in crustal heat production is partially compensated by a decrease in Q<sub>m</sub>
  - c: retrieving the heat parameters by fitting different crustal thicknesses underestimates A and overestimates Q<sub>m</sub>, this has strong implications: more weight to deeper components means larger surface footprints - more strong in d (38 mW/m² means a hot lithosphere: implications in strength)
 Take home point: non-linear effects justify more complex regression schemes. Suggestion: in the ideal case above, a uniform search in parameter space finds the correct thickness vs surface heat flow relationship. What about the real case? (near-surface "noise", uneven sampling, ...)
- The same LAB depth results in different Q<sub>m</sub> - strong control by crust and shallow structures.
- Sensitivity analysis: we must evaluate how structures of regional thermal significance can be sensed. What are the instrumental requirements?

## references

[1] Lee, W. H. K., Uyeda, S. (1965). Review of heat flow data. In Terrestrial heat flow (Vol. 8, pp. 87-190). American Geophysical Union. DOI:10.1029/GM003p0087  
 [2] Fullea, J., Alonso, J. C., Connolly, J. A. D., Fernández, M., García-Castellanos, D., Zeyen, H. (2009). LitMod3D: An interactive 3-D software to model the thermal, compositional, density, seismological, and rheological structure of the lithosphere and lithospheric upper mantle. Geochemistry, Geophysics, Geosystems, 10(8), 1-21. DOI:10.1029/2009GC002391  
 [3] Vila, M., Fernández, M., Jiménez-Munt, I. (2010). Radiogenic heat production variability of some common lithological groups and its significance to lithospheric thermal modeling. Tectonophysics, 490(3-4), 152-164. DOI:10.1016/j.tecto.2010.05.003  
 [4] Frey, J., Sippel, J., Scheck-Wenderoth, M., Bär, K., Stiller, M., Kracht, M., & Fritsche, J. G. (2015). Heterogeneous Crystalline Crust Controls the Shallow Thermal Field - A Case Study of Hesse (Germany). Energy Procedia, 76, 331-340. DOI:10.1016/j.egypro.2015.07.837  
 [5] Rudnick, R. L., McDonough, W. F., O'Connell, R. J. (1998). Thermal structure, thickness and composition of continental lithosphere. Chemical Geology, 145(3-4), 395-411. DOI:10.1016/S0009-2541(97)00151-4  
 [6] Jaupart, C., Mareschal, J.-C. (2011). Heat Generation and Transport in the Earth. Cambridge University Press. ISBN:9781139493628  
 [7] Huang, Y., Chubakov, V., Mantovani, F., Rudnick, R. L., McDonough, W. F. (2013). A reference Earth model for the heat-producing elements and associated geoneutrino flux. Geochemistry, Geophysics, Geosystems, 14(6), 2093-2029. DOI:10.1002/ggge.20129  
 [8] Lachenbruch, A. H. (1970). Crustal temperature and heat production: Implications of the linear heat-flow relation. Journal of Geophysical Research, 75(17), 3291. DOI:10.1029/JB075i017p03291  
 [9] Srivastava, K., Singh, R. N. (1998). A model for temperature variations in sedimentary basins due to random radiogenic heat sources. Geophysical Journal International, 135(3), 727-730. DOI:10.1046/j.1365-246X.1998.00693.x  
 [10] Jokinen, J., Kukkonen, I. T. (1999). Random modelling of the lithospheric thermal regime: Forward simulations applied in uncertainty analysis. Tectonophysics, 306(3-4), 277-292. DOI:10.1016/S0040-1951(99)00061-X  
 [11] Papanos, M. E., T. G. Masters, G. Laske, and Z. Ma (2014). LITHO1.0: An updated crust and lithospheric model of the Earth. J. Geophys. Res. Solid Earth, 119, 2153-2173. DOI:10.1002/2013JB010626 [12] Gosnold, W. D., 2011. Global heat flow database, Provided by the University of North Dakota, interim custodian (www.heatflow.usund.edu/index2.html)  
 [12] Braitenberg, C., Wienecke, S., Ebbing, J., Born, W., Redfield, T. (2007). Joint Gravity and Isostatic Analysis for Basement Studies - A Novel Tool. EGM 2007 International Workshop, Innovation in EM, Grav and Mag Methods: A New Perspective for Exploration, 16-18

\*contact: alberto.pastorutti@phd.units.it  
Tectonophysics & Geodynamics Research Group  
Dept. of Mathematics and Geosciences, Univ. of Trieste  
via Edoardo Weiss, 1 34128 Trieste (Italy)

Acknowledgements:  
work by author AP is being supported by a grant under the European Social Fund, through resources of Region Friuli Venezia Giulia in the form of a PhD fellowship at the University of Trieste. POR 2014-2020, HeA UNITS op. 3, 25/15, FP1687011001  
The contents expressed herein are those of the authors and do not necessarily reflect the views of the funding entities.