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Error Characteristics of Satellite-only Global Gravity Models after Solid Earth Data Reductions

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Background

- Global satellite-only gravity models provide unparalleled spatial homogeneity in coverage and quality, at length scales suitable for lithospheric density modelling.
- Geophysical inverse problems require isolating an anomalous signal in the observed gravity field, through removal of the effect of known masses (data reduction, e.g. topography, sediments ...)
- Error characteristics of gravity models: 3 orders of magnitude smaller than reduction uncertainty at the same length scales. Data reduction and inversion parameters are the main error sources.

Forward Modelling Algorithm

We rely on the **SHTOOLS** [1] implementation of **Wieczorek & Phillips (1998)** algorithm [2] spectral forward modelling algorithm for the **potential of a relief with lateral variations of density,** referenced to a spherical interface.

from Wieczorek (2007) [3]

Sph. harmonics coeffients of the relief (to the n-th power)

$$(\rho h^n)_{lm} = \frac{1}{4}\int_{\Omega} \left[\rho(\theta,\phi)h^n(\theta,\phi)\right] Y_{lm}(\theta,\phi)d\Omega \quad \textit{with} \quad \Omega = (\theta,\phi) \qquad (1)$$

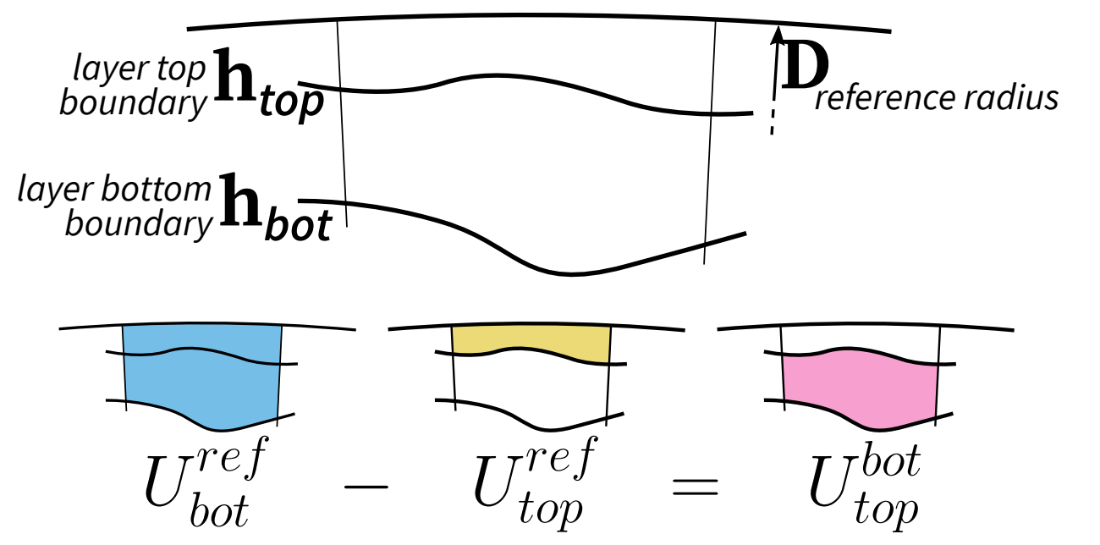
Sph. harmonics coefficients of potential

$$C_{lm} = \frac{4\pi D^3}{M(2l+1)} \sum_{n=1}^{n_{max}} \frac{(\rho h^n)_{lm}}{D^n\ n!} \frac{\prod_{j=1}^{n}(l+4-j)}{(l+3)} \qquad (2)$$

Potential (exterior to relief)

$$U(\mathbf{r}) = \frac{GM}{r}\sum_{l=0}^{l_{max}}\sum_{m=0}^{l}\left(\frac{R_0}{r}\right)^l C_{lm}Y_{lm}(\Omega) \qquad (3)$$

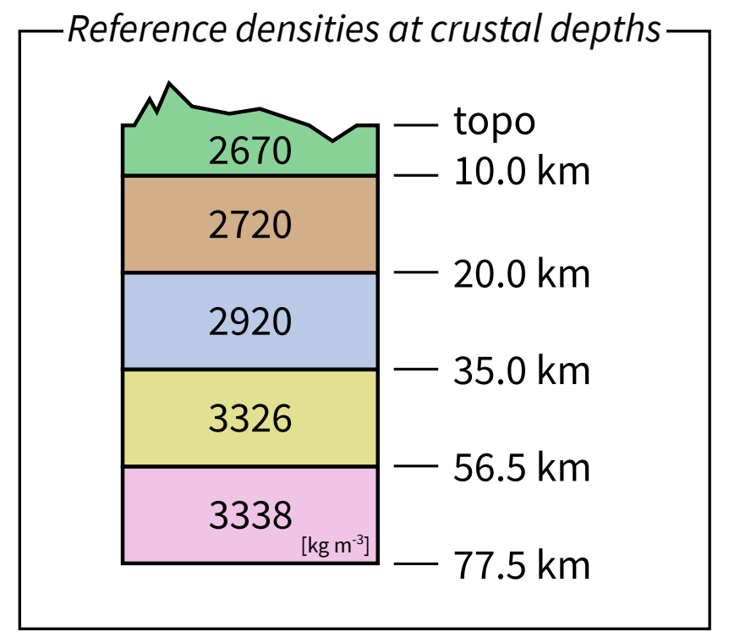
We set up a **layer-wise forward modelling** scheme:



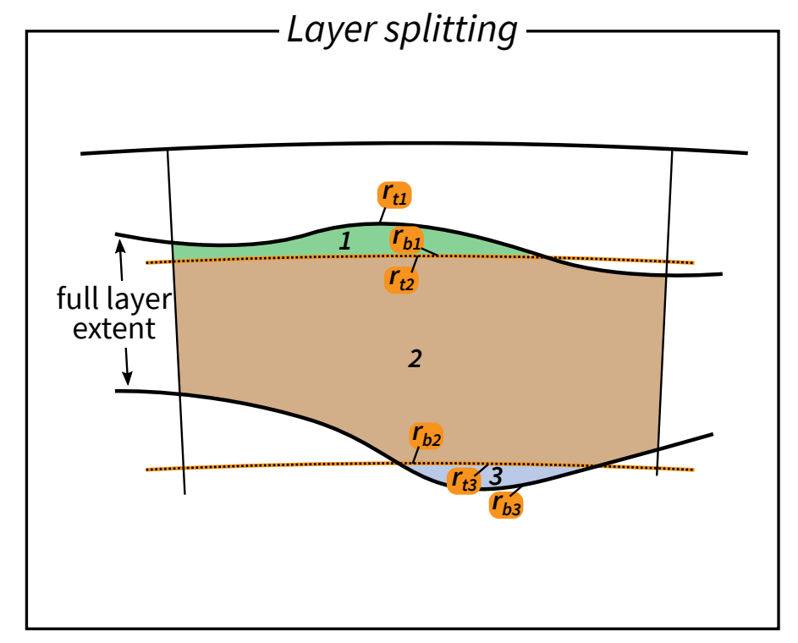
Density reference and layer splitting

Global density reference: adapted from **AK135**[4], discretized in geocentric **ellipsoidal shells** of constant density. The "known densities" of the modelled layers are expressed against this reference, after slicing each layer according to the shells it intersects.

Reference densities at crustal depths



Layer splitting



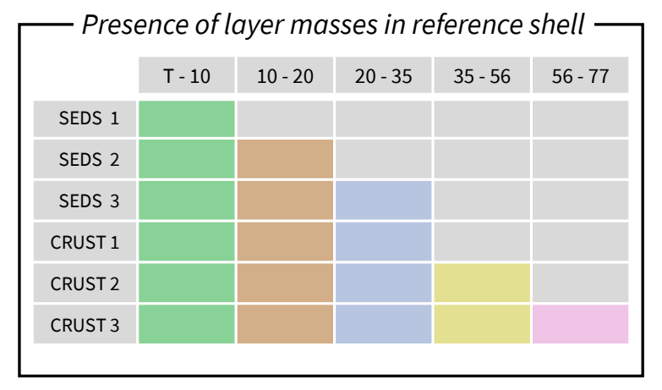
Terrain correction: input topography, water, ice

We use the **Earth2014**, 1 arc-min shape model [5] to obtain a terrain correction (TC). We forward modelled an **ellipsoid-referenced** solid topography effect, plus water and ice stripping. When this TC is removed from the observed gravity disturbance, we obtain "No Ellipsoidal Topography of Constant density" gravity disturbance (**NETC**, see [6]).

Sub-surface data: LITHO1.0 [7]

- Readily available, global depth-density model, layer defined: topography to lithosphere-asthenosphere boundary.
- Surface wave based, from an integrated starting model (multiple sources): no information on coverage and data uncertainty, this suggests caution.
- We consider it **fit-for-purpose** for this uncertainty-propagation test.

Presence of layer masses in reference shell

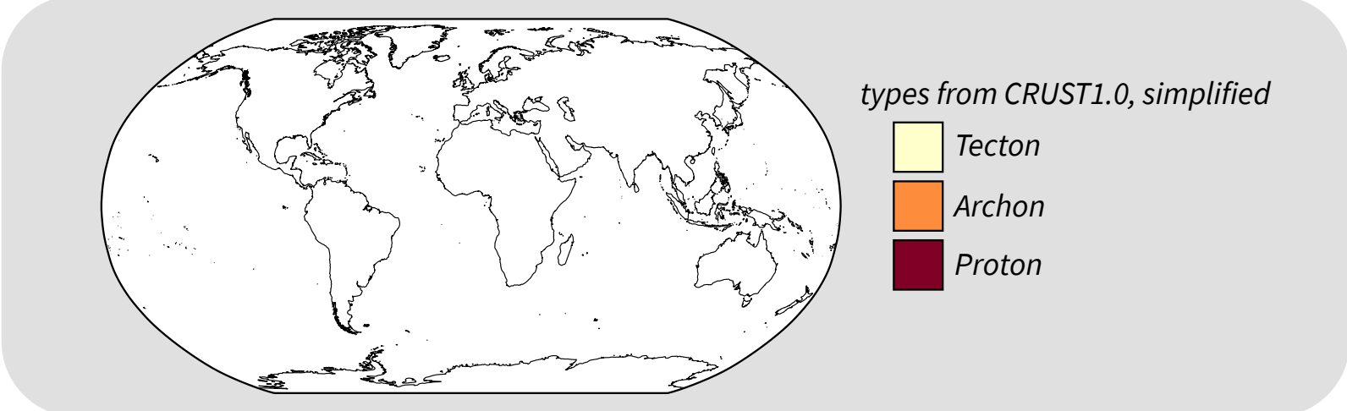


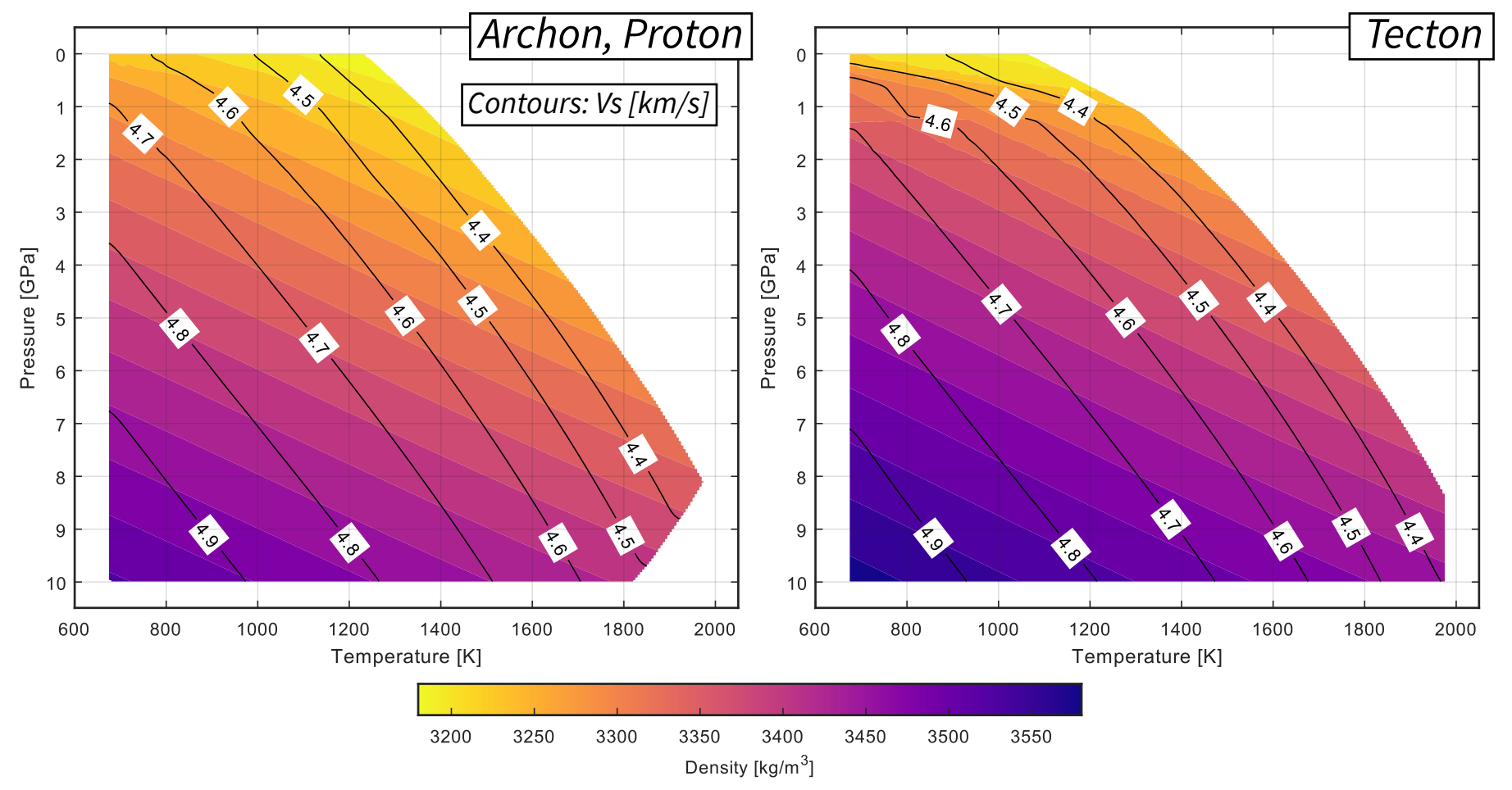
	T - 10	10 - 20	20 - 35	35 - 56	56 - 77
SEDS 1	🟩				
SEDS 2	🟩	🟫			
SEDS 3	🟩	🟫	🟦		
CRUST 1	🟩	🟫	🟦		
CRUST 2	🟩	🟫	🟦	🟨	
CRUST 3	🟩	🟫	🟦	🟨	🟪

Data extraction: from the 1 arc-degree tessellated LITHO1.0 to a regular 0.25° x 0.25° global grid, then to the spherical harmonics coefficents of eq. 1. We perform the triangulation+interpolation **using StriPy** [8]. **Depth reference**: we tie Earth2014 bedrock (provided as geocentric radius) to LITHO1.0 top of first sediment layer. Depths are thus provided as spherical radii.

Lithospheric mantle: velocity-to-density conversion

- Vs to density for LITHO1.0 'LID' layer (Moho to LAB)
- density and Vs forward modelling using Perple_X [9]
- simple compositional model: Archon/Tecton, according to Griffin et al. 2009 [10]





Uncertainty propagation through random modelling

- error assumptions on the input data: **depth** and **density**
- random modelling on 5000 independent draws

depth uncertainty, st. dev	density uncertainty, st. dev
5 % of depth	100 kg m^{-3}

!

- simple error criteria (realistic, but no spatial variability)
- no error covariance information is included: each node assumed independent
- criteria for 5000 draws: high enough to observe power-law decay in error degreee variances (i.e. effect of assuming uncorellated errors attenuates)

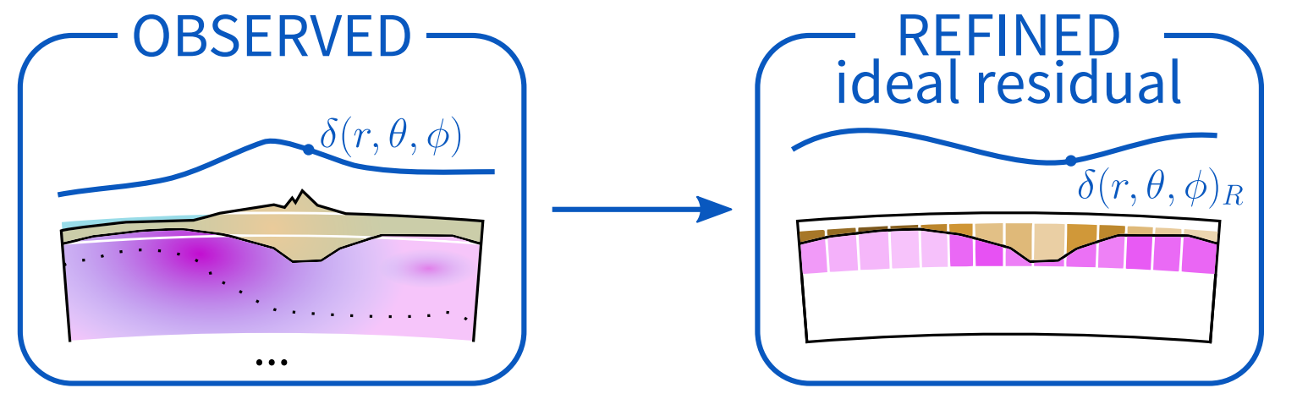
Implementation:

- parallel implementation, using the *multiprocessing* Python module [11]
- 3,3 seconds per sliced-layer, per worker, per draw (e.g. 2 hours on 40 workers)
- random draws are partitioned in 100-draws blocks
- the variance of g partitions of k draws is consolidated, using the following:

$$\mathrm{Var}(X_1,\cdots,X_{gk}) = \frac{k-1}{gk-1}\left(\sum_{j=1}^{g}V_j + \frac{k(g-1)}{k-1}\mathrm{Var}(E_j)\right) \quad \text{with } E_j, V_j \text{ mean and variance of each partition}$$

→ could be easily scaled to more complex, larger schemes

Ideal output: result of unmodelled masses



- no topography
- homogenous crust
- nomogenous mantle

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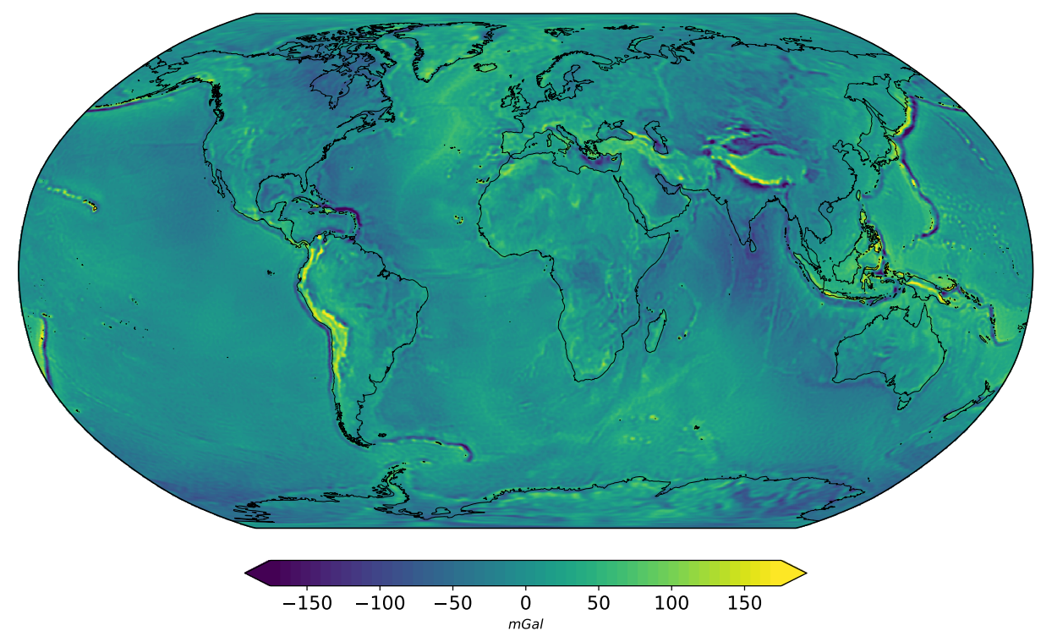
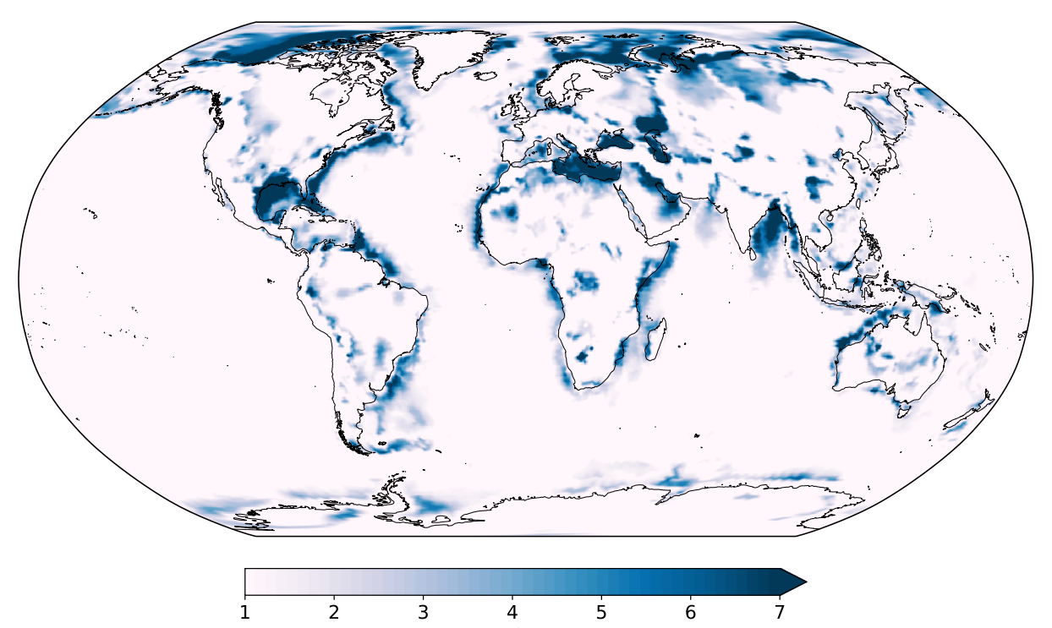
Results: forward modelled reductions

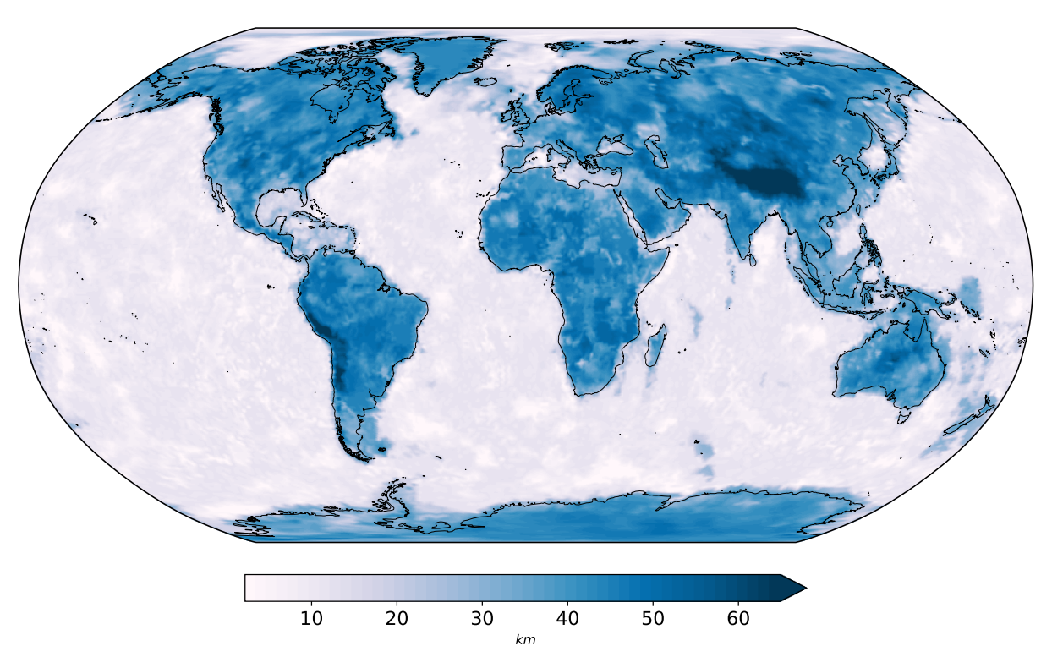
All functionals were computed at 10 km over GRS80, up to SH degree = 280

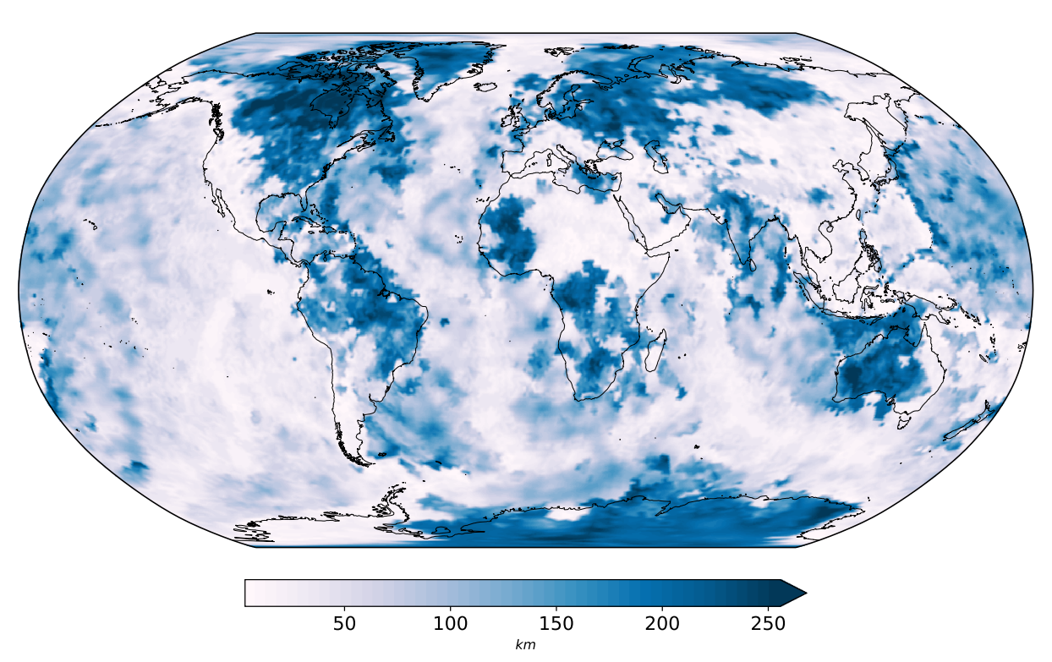
input GGM

GOCO05s, δg
GGM gravity disturbance





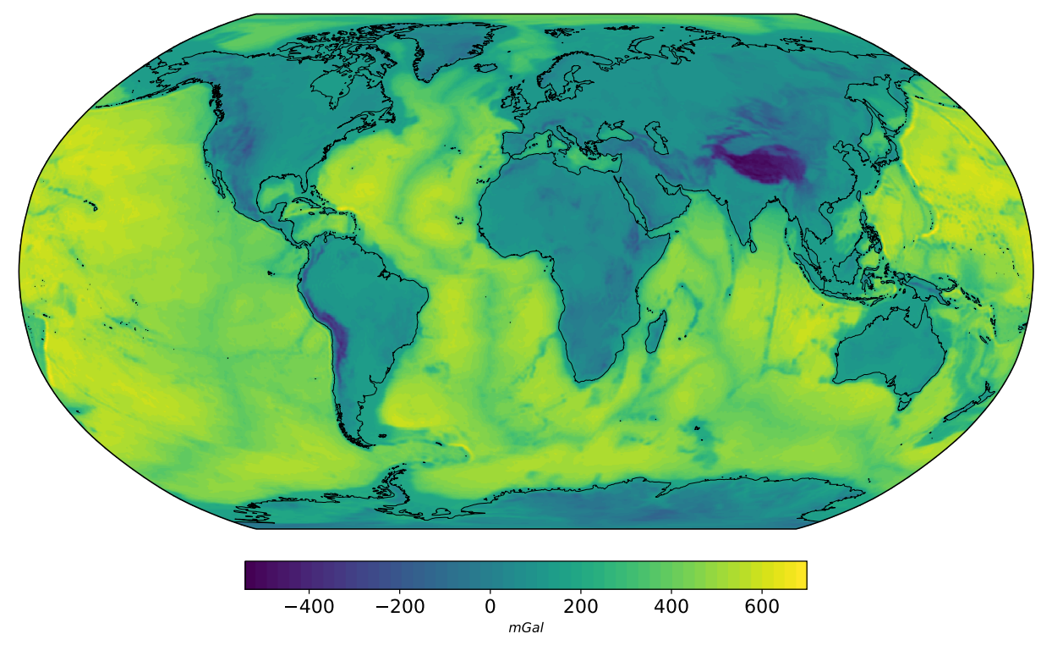




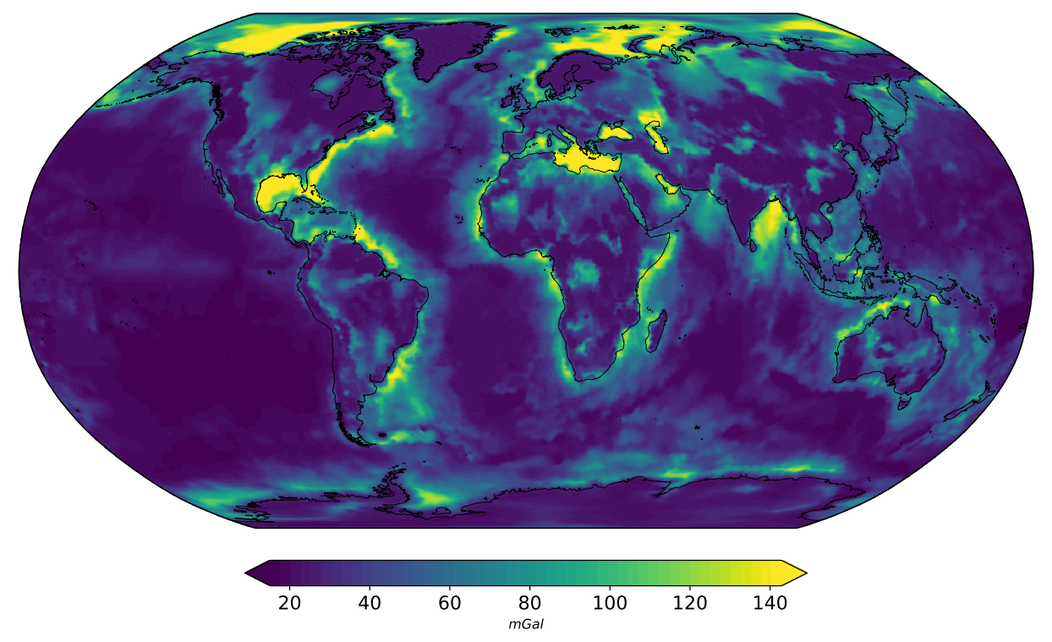
cumulative thickness

correction due to modelled contrasts

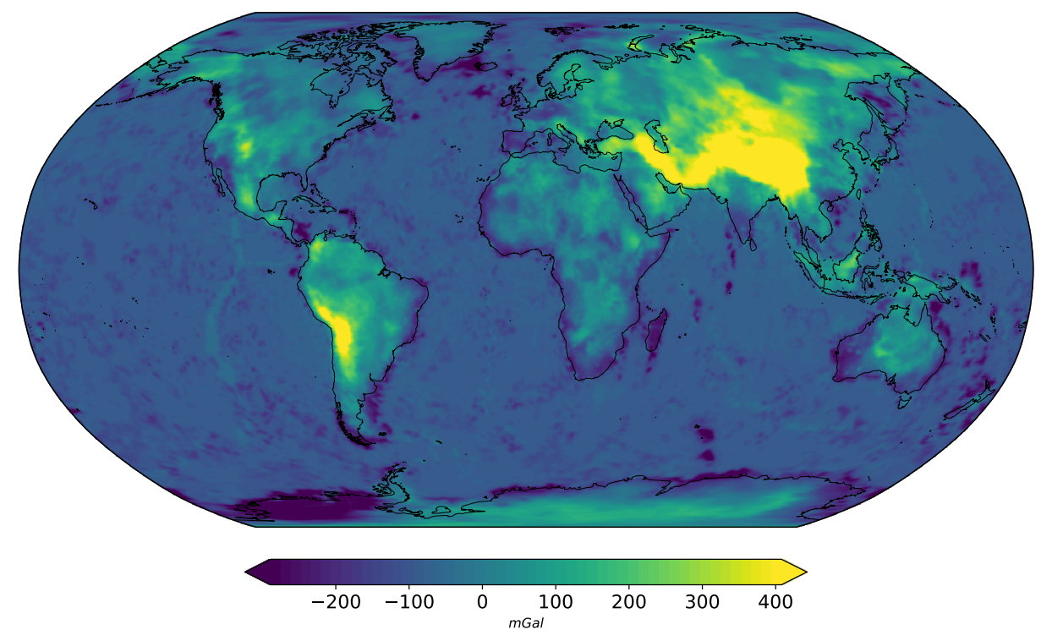
TC, g_r
terrain correction



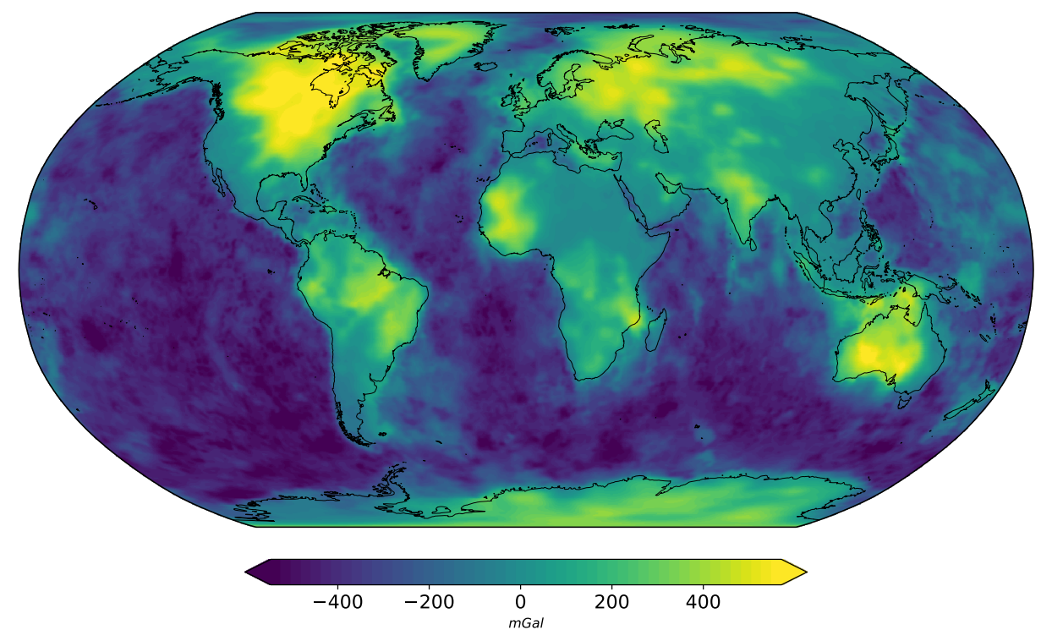
SC, g_r
sediment correction



CC, g_r
crystalline crust correction

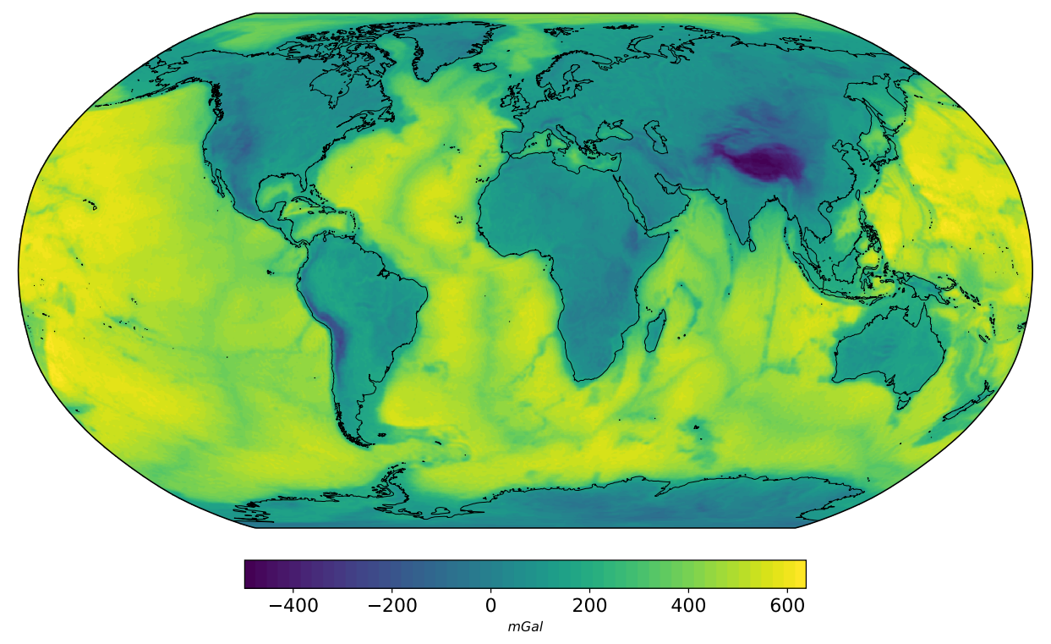


LIDC, g_r
lithospheric mantle (LID) correction

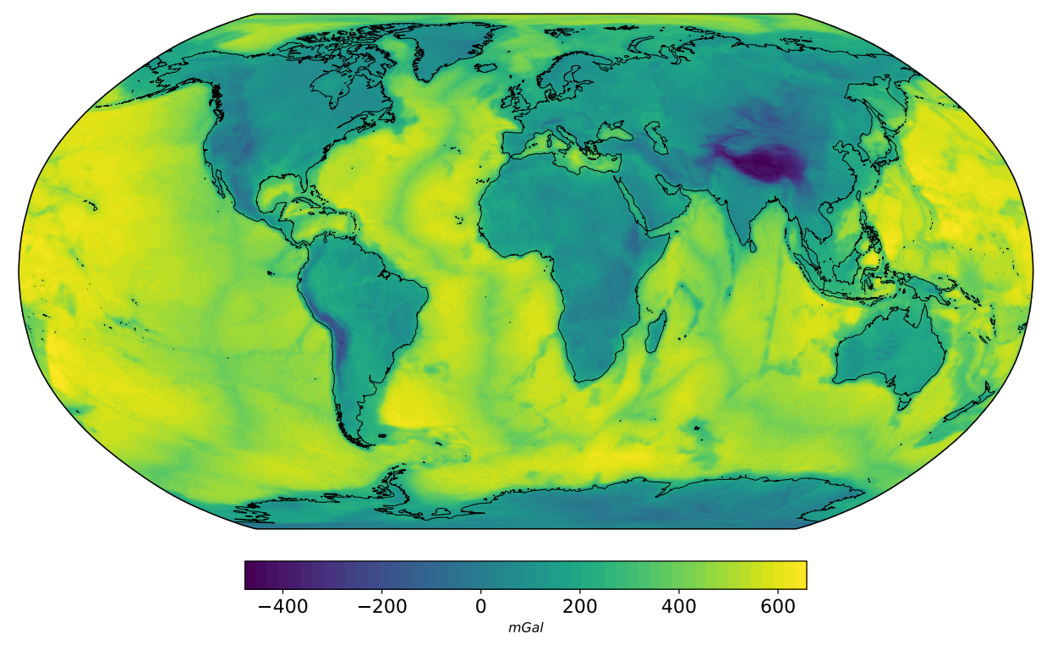


refined gravity disturbance
$\delta g = |\nabla W_{GGM} + \nabla W_m| - |\nabla U|$

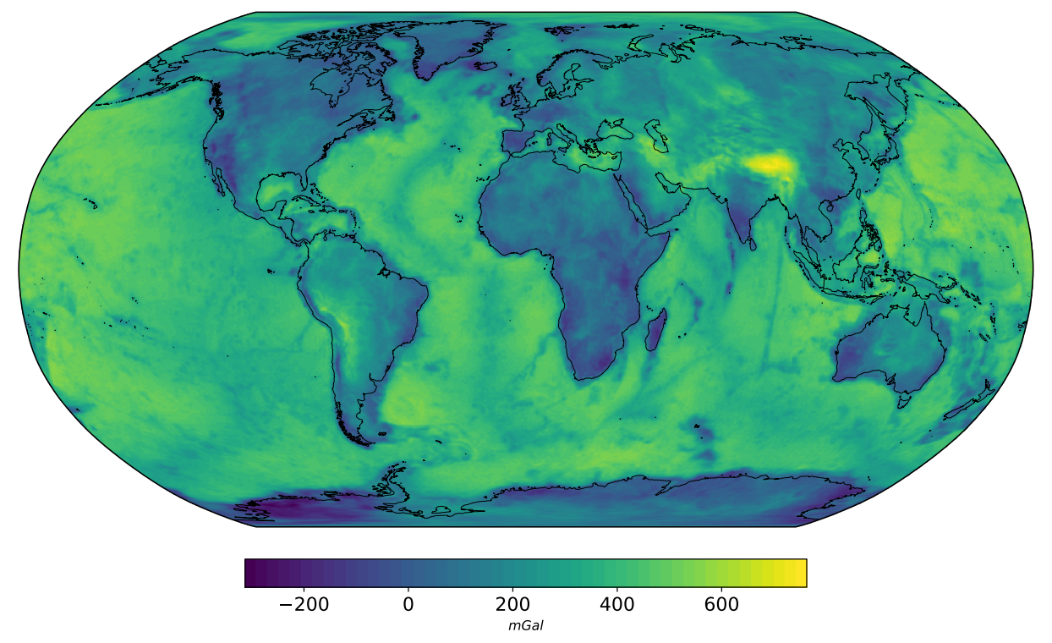
NETC, δg
No Ellipsoidal Topography of Constant density



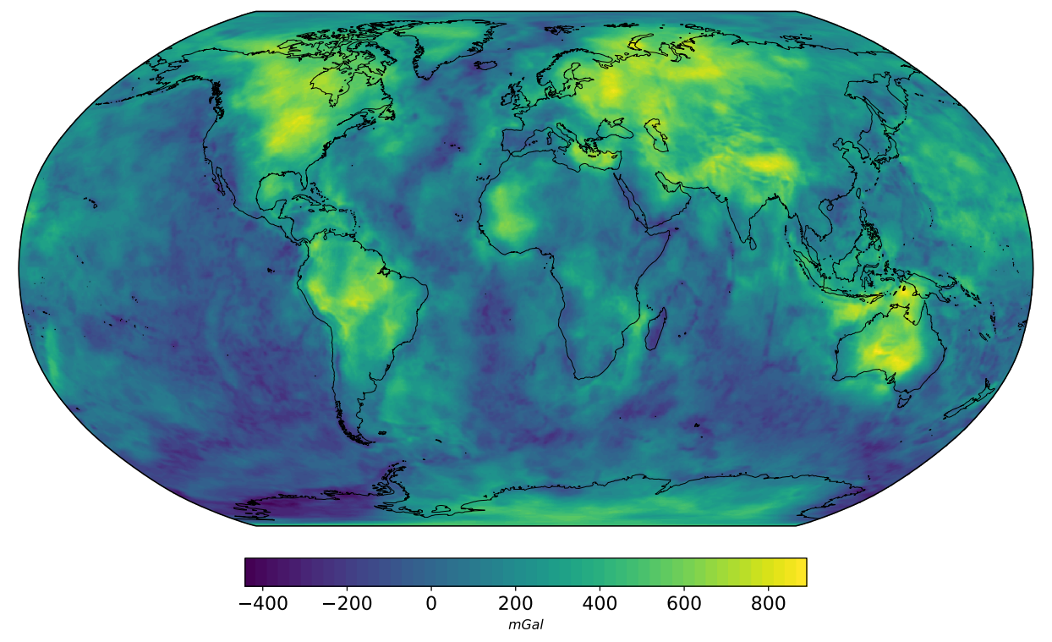
SC NETC, δg
sediments corrected NETC disturbance



FC NETC, δg
whole-crust (seds+crust) corrected NETC disturbance

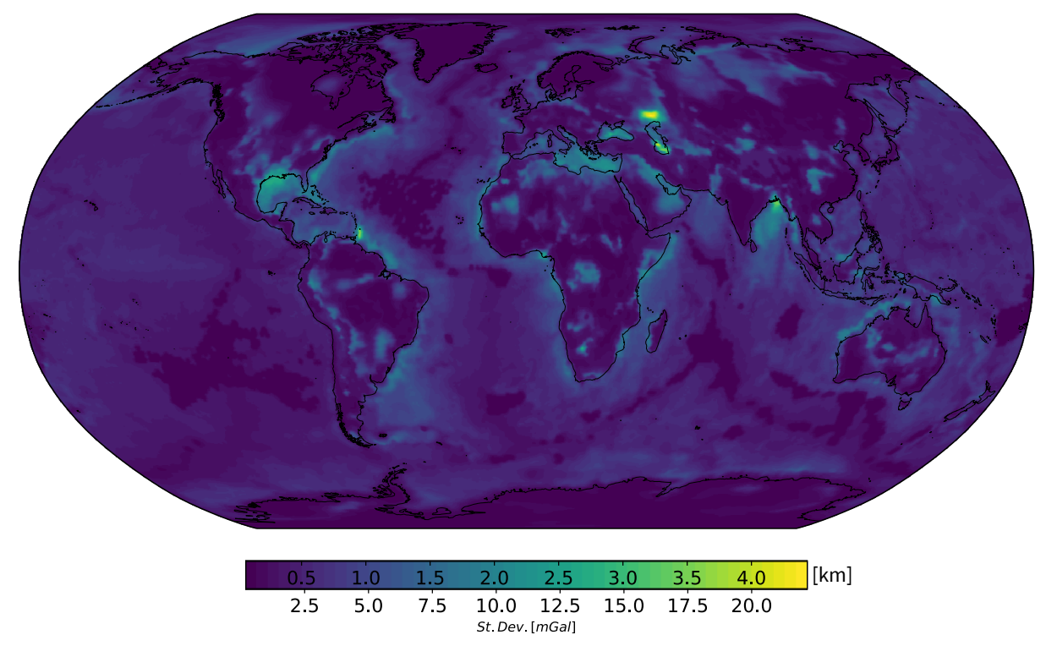


LC NETC, δg
whole-lithosphere corrected NETC disturbance

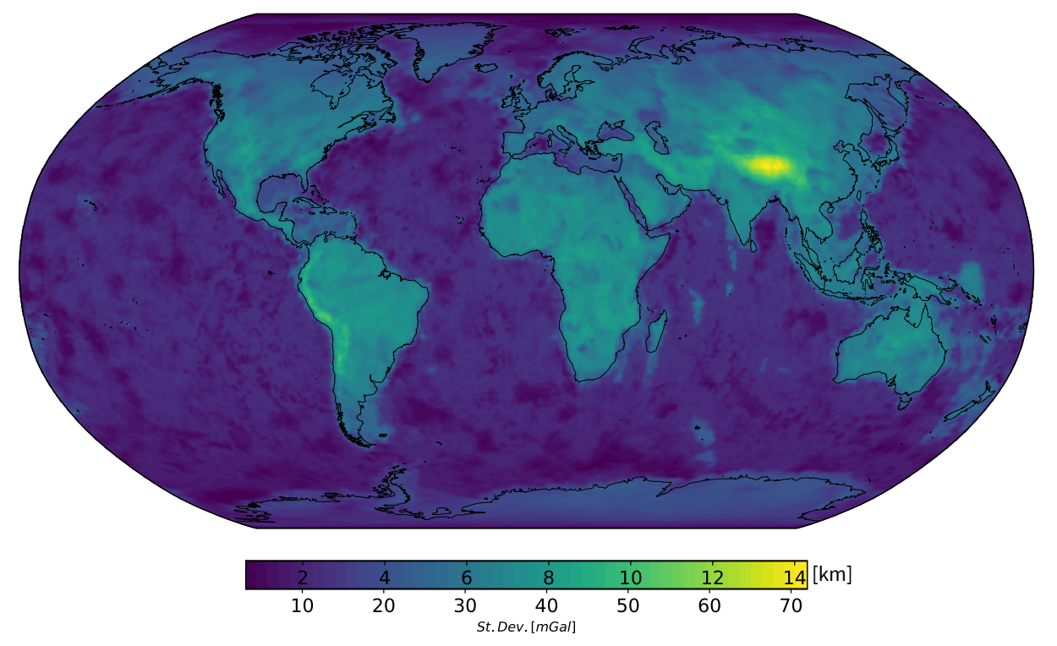


correction uncertainty
and equivalent Moho uncertainty
(±2σ error bars) 485 kg m^{-3}

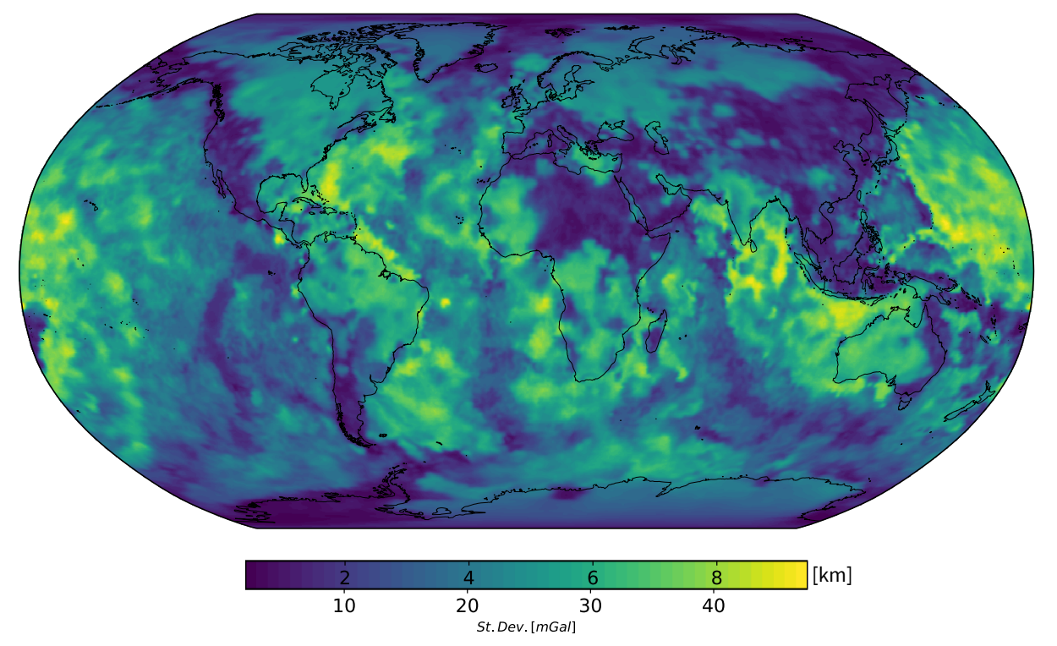
SC, σ(g_r)
Sediment correction, standard deviation



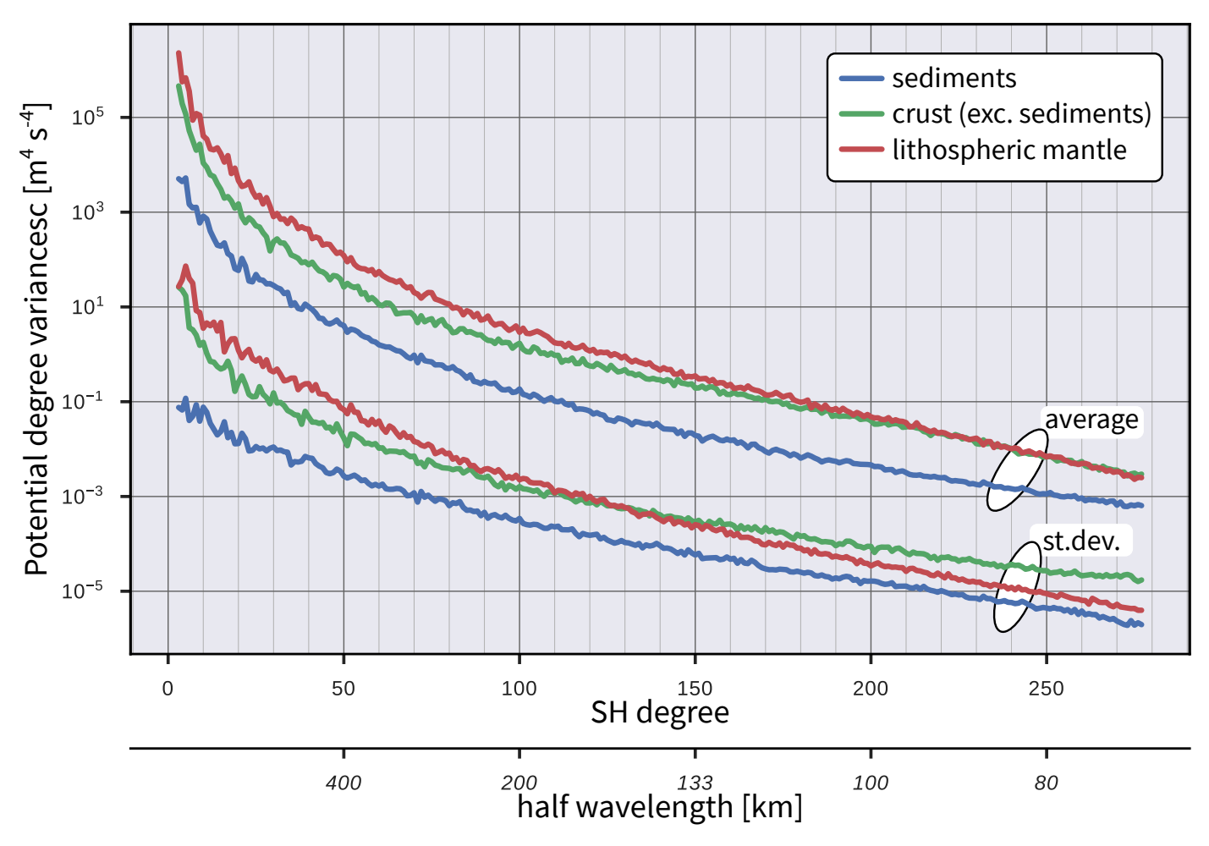
CC, σ(g_r)
whole-CC correction, standard deviation



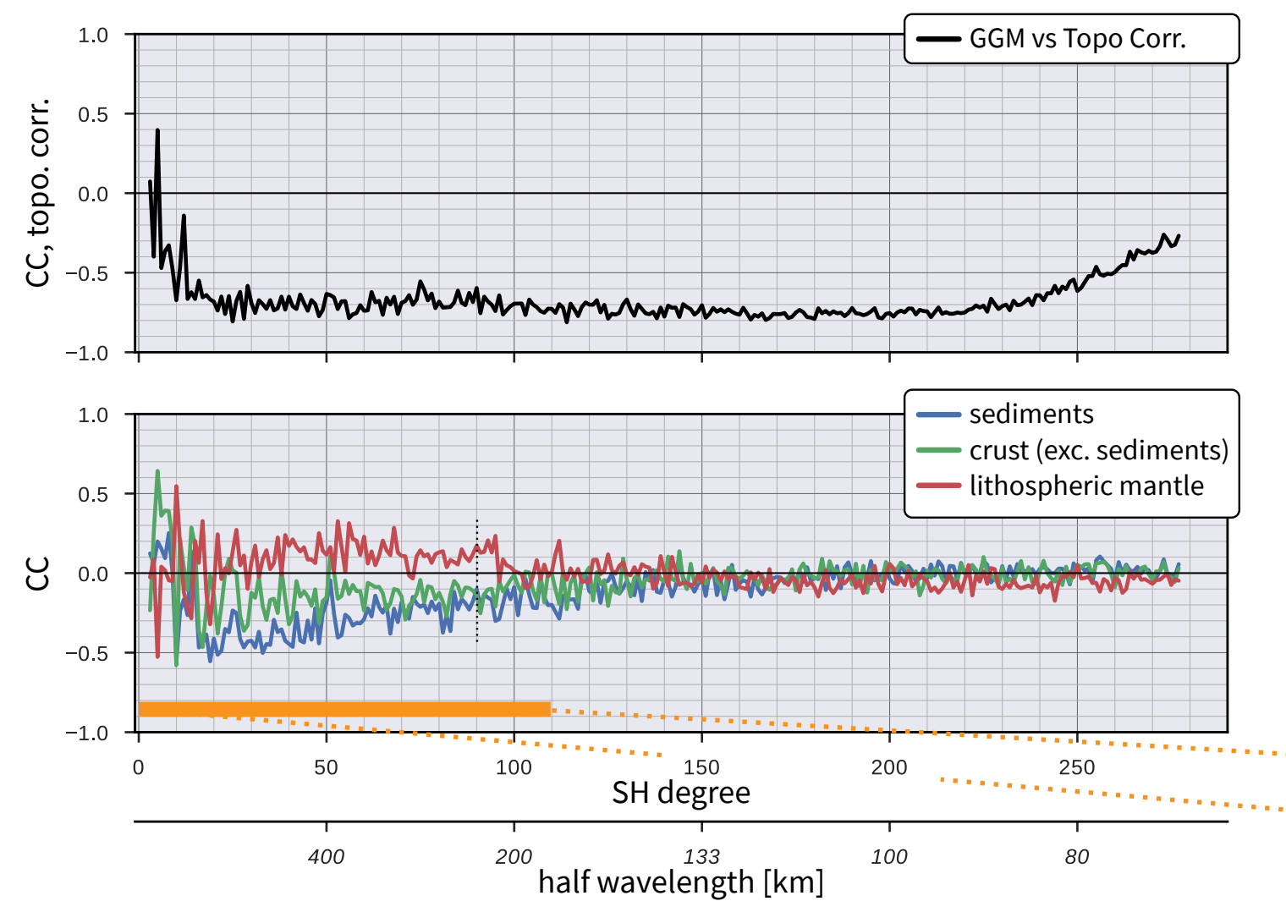
LIDC, σ(g_r)
LID correction, standard deviation



degree variances spectra: modelled effect against error



correlation coefficients, against GGM



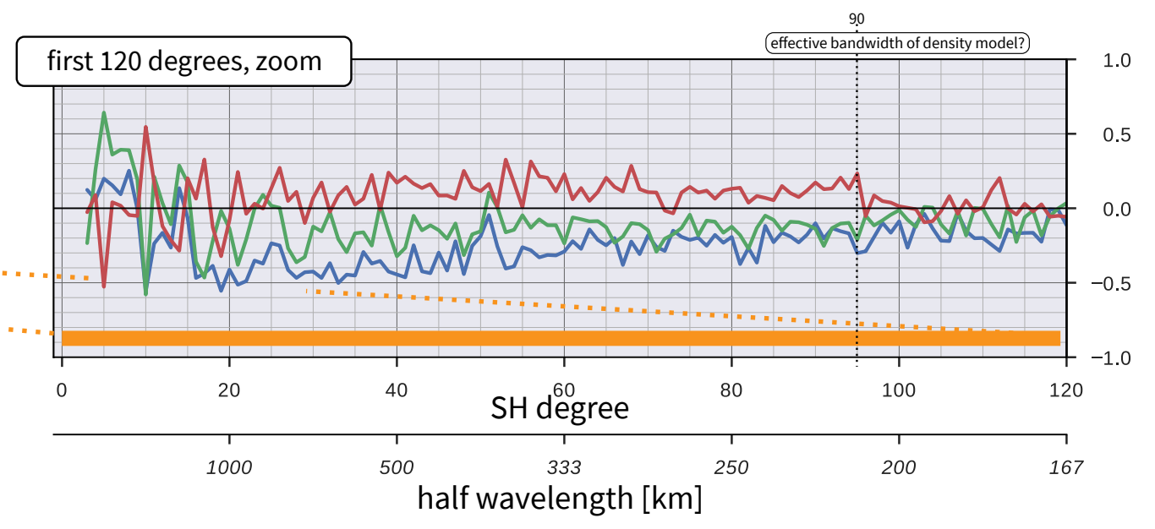
Correlation coefficients were computed according to Wieczorek [3] formulation. For two SH functions f and g, for a given degree l:

$$CC_{fg}(l) = \frac{Sfg(l)}{\sqrt{S_{ff}(l)\ S_{fg}(l)}}$$

where S_{ff}, S_{gg} are the power spectra of the functions (degree variance) and S_{fg} is the cross-power spectrum (degree covariance).

$$S_{ff}(l) = \sum_{m=-l}^{l} f^2_{lm} \qquad S_{gg}(l) = \sum_{m=-l}^{l} g^2_{lm} \qquad S_{fg}(l) = \sum_{m=-l}^{l} f_{lm}\, g_{lm}$$

CC(l) can possess values between 1 and -1.



concluding remarks

Outcome (and collaterals):

- error estimate of gravity model "after reduction"
  relying on a layer-based, spectral domain forward modelling of reductions and propagated errors
- reproduction of "common" and "novel" reductions
  from a topography-free to an (ideally) lithosphere-free disturbance (albeit with simple assumptions!)

Room for improvement:

- realistic, data dependent, error estimates
  e.g. weight according to data density, observable - and error propagation from conversion
- upper mantle model and velocity conversion
  integrate available models, removal of lithosphere only shows LAB as artifact compositional model: refine or assess effect of "coarse" assumptions?
- gravity-model-aware adaptation of reductions
  truncating at maximum SH degree is not enough
  e.g. take into account high-degree regularization of sat-only models

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