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# Uncertainty of Satellite-gravity-derived Moho Estimates: Contribution of Data Reductions

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## Background

- Global satellite-only gravity models provide unparalleled spatial homogeneity in coverage and quality, at length scales suitable for lithospheric density modelling.
- Geophysical inverse problems require isolating an anomalous signal in the observed gravity field, through removal of the effect of known masses (data reduction, e.g. topography, sediments ... )
- Error characteristics of gravity models: 3 orders of magnitude smaller than reduction uncertainty at the same length scales. Data reduction and inversion parameters are the main error sources.

## Forward Modelling Algorithm

We rely on the **SHTOOLS** [1] implementation of **Wieczorek & Phillips (1998)** algorithm [2] spectral forward modelling algorithm for the **potential of a relief with lateral variations of density,** referenced to a spherical interface.

*from Wieczorek (2007) [3]*

Sph. harmonics coeffients of the relief (to the n-th power)

$$(\rho h^n)_{lm} = \frac{1}{4}\int_{\Omega} \left[\rho(\theta,\phi)h^n(\theta,\phi)\right] Y_{lm}(\theta,\phi)d\Omega \quad \textit{with} \quad \Omega = (\theta,\phi) \tag{1}$$

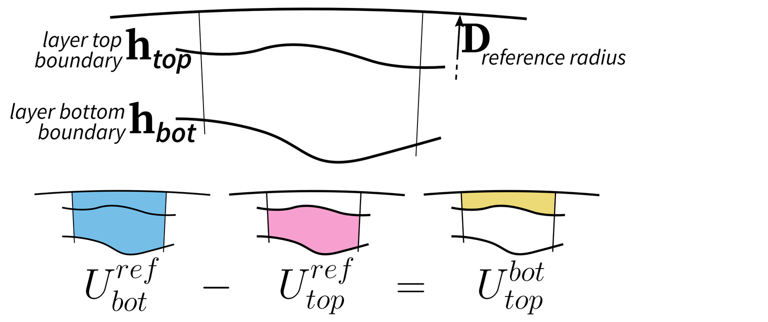
Sph. harmonics coeffients of potential

$$C_{lm} = \frac{4\pi D^3}{M(2l+1)} \sum_{n=1}^{n_{max}} \frac{(\rho h^n)_{lm}}{D^n\, n!} \frac{\prod_{j=1}^{n}(l+4-j)}{(l+3)} \tag{2}$$

Potential (exterior to relief)

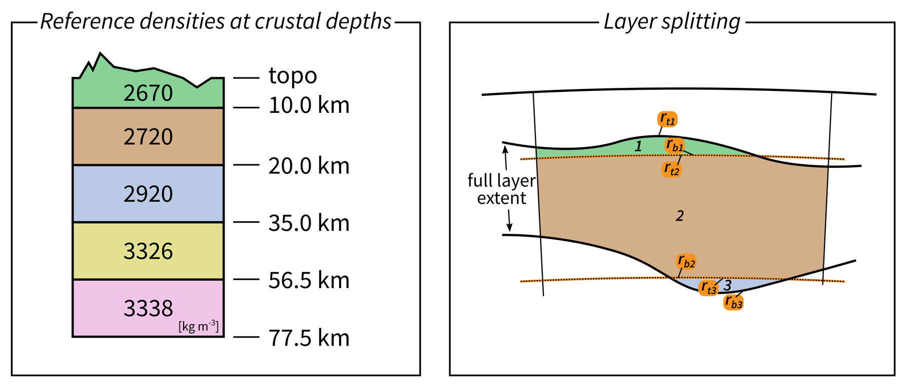
$$U(\mathbf{r}) = \frac{GM}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{R_0}{r}\right)^l C_{lm}Y_{lm}(\Omega) \tag{3}$$

We set up a **layer-wise forward modelling** scheme:




## Density reference and layer splitting

Global density reference: adapted from **AK135**[4], discretized in geocentric **ellipsoidal shells** of constant density. The "known densities" of the modelled layers are expressed against this reference, after slicing each layer according to the shells it intersects.

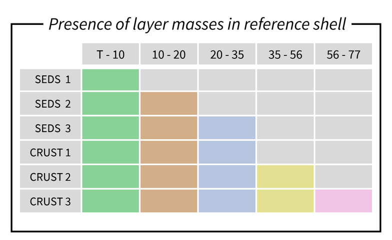



## Terrain correction: input topography, water, ice

We use the **Earth2014**, 1 arc-min shape model [5] to obtain a terrain correction (TC). We forward modelled an **ellipsoid-referenced** solid topography effect, plus bathymetry and ice stripping. When this TC is removed from a the observed gravity disturbance, we obtain "No Ellipsoidal Topography of Constant density" gravity disturbance (**NETC**, see [6]).

## Sub-surface data: LITHO1.0 [7]

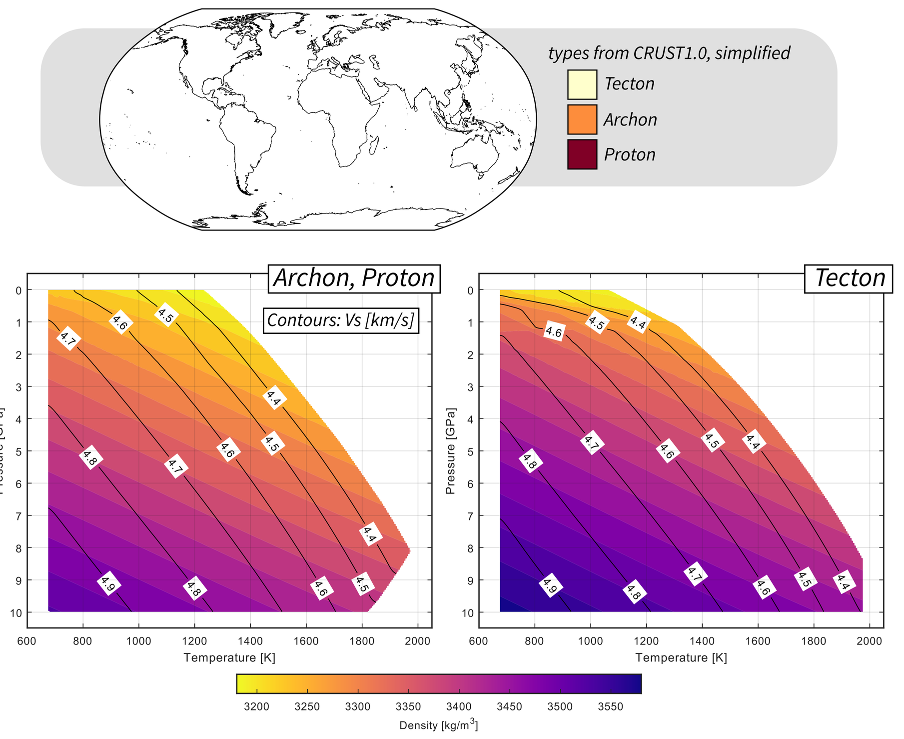
- Readily available, global depth-density model, layer defined: topography to lithosphere-asthenosphere boundary.
- Surface wave based, from an integrated starting model (multiple sources): no information on coverage and data uncertainty, this suggest caution.
- We consider it **fit-for-purpose** for this uncertainty-propagation test.




**Data extraction**: from the 1 arc-degree tassellated LITHO1.0 to a regular 0.25° x 0.25° global grid, then to the spherical harmonics coefficents of eq. 1. We perform the triangulation+interpolation **using StriPy** [8]. **Depth reference**: we tie Earth2014 bedrock (provided as geocentric radius) to LITHO1.0 top of first sediment layer. Depths are thus provided as spherical radii.

## Lithospheric mantle: velocity-to-density conversion

- Vs to density for LITHO1.0 'LID' layer (Moho to LAB)
- density and Vs forward modelling using Perple_X [9]
- simple compositional model: Archon/Tecton, according to Griffin et al. 2009 [10]




## Uncertainty propagation through random modelling

- error assumptions on the input data: **depth** and **density**
- random modelling on 5000 independent draws

| depth uncertainty, st. dev | density uncertainty, st. dev |
|---|---|
| 5 % of depth | 100 kg m⁻³ |

!
- simple error criteria (realistic, but no spatial variability)
- no error covariance information is included: each node assumed independent
- criteria for 5000 draws: high enough to observe power-law decay in error degreee variances (i.e. effect of assuming uncorellated errors attenuates)

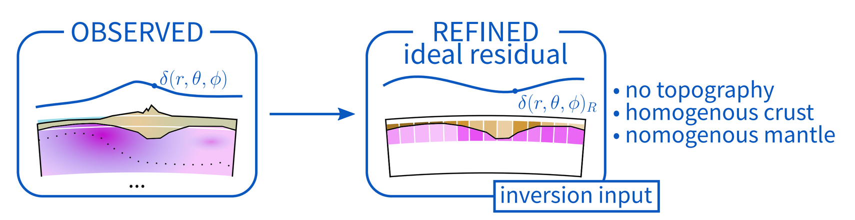
*Implementation:*
- parallel implementation, using the *multiprocessing* Python module [11]
- 3,3 seconds per sliced-layer, per worker, per draw (e.g. 2 hours on 40 workers)
- random draws are partitioned in 100-draws blocks
- the variance of g partitions of k draws is consolidated, using the following:

$$\mathrm{Var}(X_1,\cdots,X_{gk}) = \frac{k-1}{gk-1}\left(\sum_{j=1}^{g} V_j + \frac{k(g-1)}{k-1}\mathrm{Var}(E_j)\right)$$ with $E_j, V_j$ mean and variance of each partition

→ could be easily scaled to more complex, larger schemes

## From disturbance-uncertainty to Moho-uncertainty

- aim: uncertainty expressed in "equivalent Moho depth"

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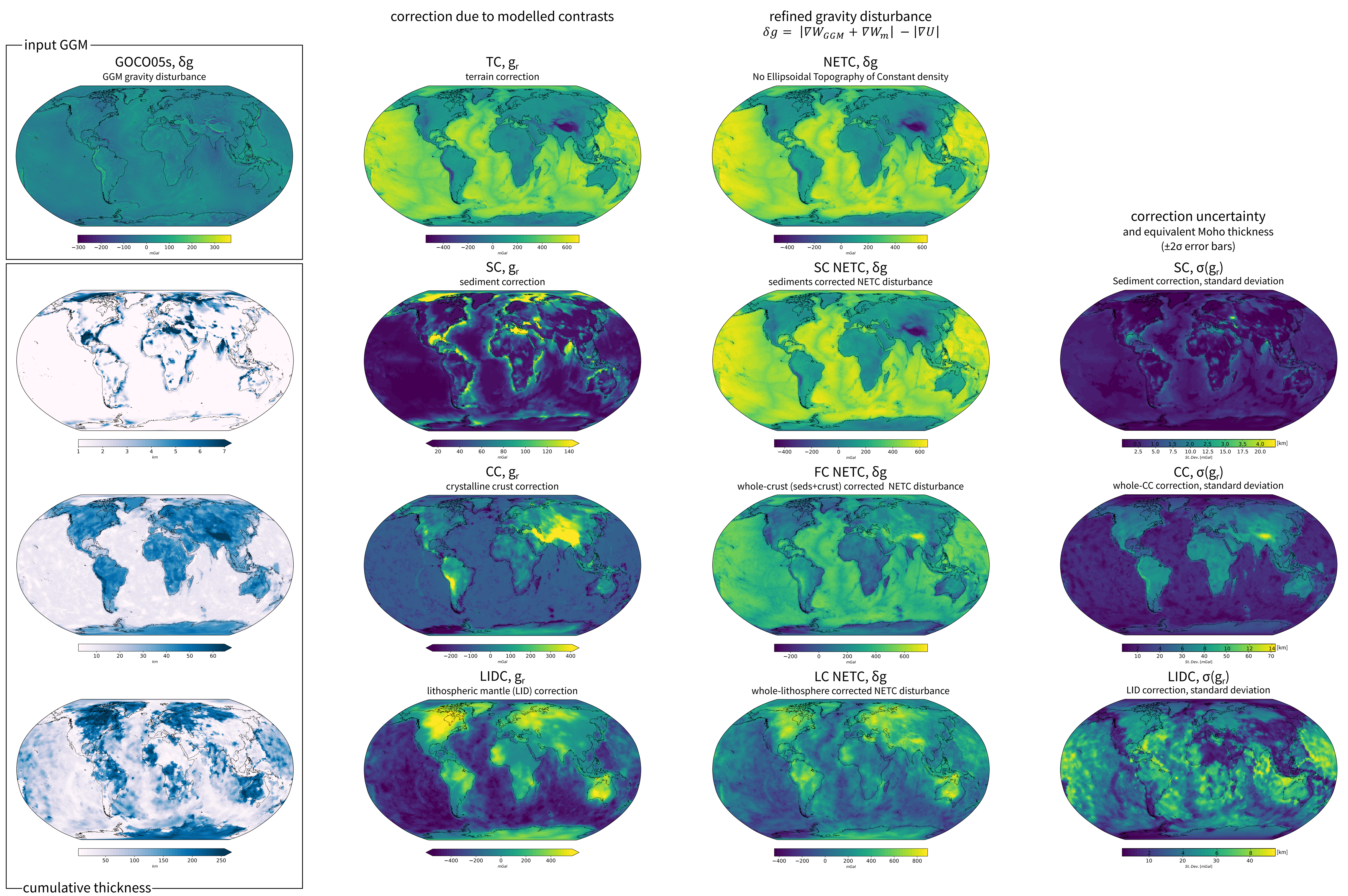
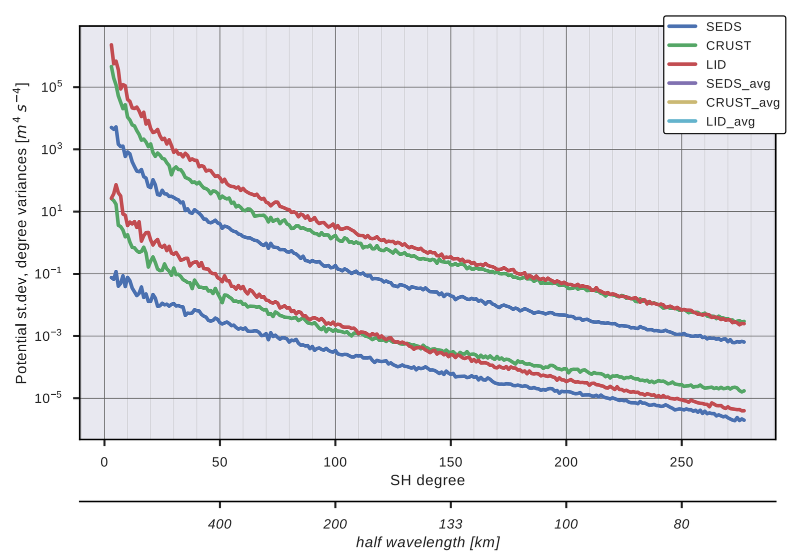
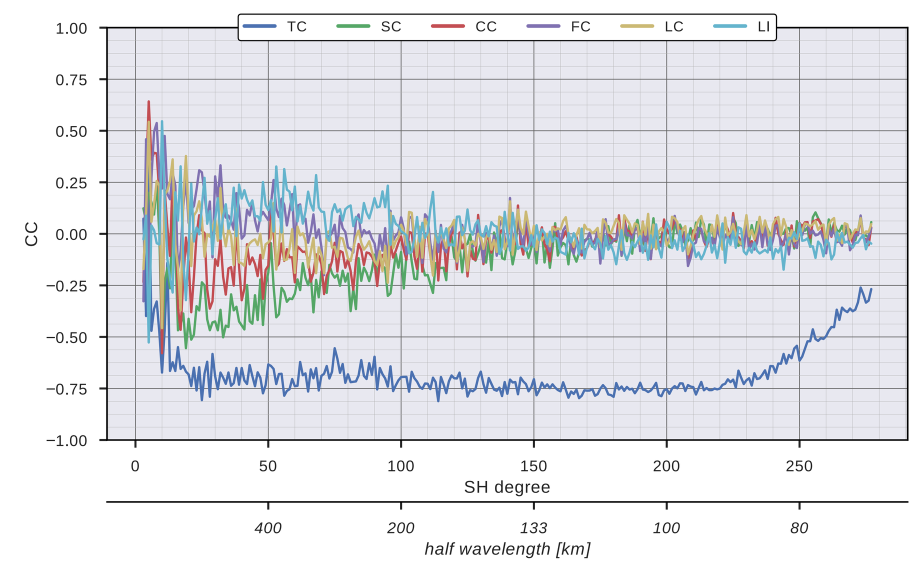
## Results: forward modelled reductions

All functionals were computed at 10 km over GRS80, up to SH degree = 280




### concluding remarks

**Outcome (and collaterals):**
- error estimate of gravity model "after reduction"
  relying on a layer-based, spectral domain forward modelling of reductions and propagated errors
- reproduction of "common" and "novel" reductions
  from a topography-free to an (ideally) lithosphere-free disturbance (albeit with simple assumptions!)

**Room for improvement:**
- realistic, data dependent, error estimates
  e.g. weight according to data density, observable - and error propagation from conversion
- upper mantle model and velocity conversion
  integrate available models, removal of lithosphere only shows LAB as artifact
  compositional model: refine or assess effect of "coarse" assumptions?
- gravity-model-aware adaptation of reductions
  truncating at maximum SH degree is not enough
  e.g. take into account high-degree regularization of sat-only models

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