Polarization and Total Energy Spectra of the Eigenvibrations of the Earth Recorded at Trieste

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Summary
The NS and EW components of the horizontal pendulums records of the eigenvibrations of the Earth excited by the Chilean earthquake of 1960 May 22 have been analysed by using spectral and co-spectral methods in order to obtain statistical parameters describing the polarization and the total energy associated with the individual modes. The analysis is carried out on five successive partly overlapping sections of the records, thus obtaining information on the evolution of the phenomenon.

1. The free oscillations of the Earth excited by the Chilean earthquake on 1960 May 22 have been clearly recorded for the duration of 82 hours by the two horizontal EW and NS pendulums in the Grotta Gigante near Trieste (Marussi 1959, 1960). Two spectral analyses of the entire record of the NS component (the EW record was not analysed at this time) were carried out previously using a sampling interval of 2 minutes and an elementary frequency band of about $2.5 \times 10^{-2}$ c/min (Bolt & Marussi 1962) and of about $8.4 \times 10^{-4}$ c/min (Bolt 1963).

The purpose of the present research is to extend the analysis of the records to both components, dividing them into five partially overlapping 30-hour sections. The analysis of both components leads to an estimate of the total energy associated with the individual modes, and of their polarization; the division in successive sections allows the stability of the spectral peaks and of the dissipation of energy in time to be examined.

2. Extensive use of power spectral and cross-spectral analyses of the records has been made rather than of direct Fourier transforms because of the considerable saving in machine time involved and of the greater stability of the spectra thus obtained.

In principle, auto- and cross-correlation methods can be used only if the time series to be analysed are stationary and ergodic; yet the methods may be applied to time series which strictly speaking are not even stationary, as is certainly the case for time series representing isolated evolving phenomena like earthquake records. In such cases it is advisable to subdivide the record into various sections, each one short enough for the damping to be negligible, and to analyse them separately. This procedure has the further advantage of allowing for possible phase shifts in the individual harmonic components that would otherwise affect the calculation of the auto- and cross-correlation functions, thus obscuring the final results.

In our case the free oscillations of the Earth excited by the Chilean earthquake, die down almost completely in the course of the 82 hours under consideration, as a consequence of the dissipation of energy, and shifts in the phases are possible as a consequence of successive shocks at the origin and of resonance effects due to irregularities in the mantle.
For these reasons the 82 hours of recording have been subdivided into sections of 30 h duration, with overlaps of about 17 h, the centres of the successive sections being thus spaced by about 13 h.

By using such subdivisions it is furthermore possible:

(1) to check the stability in time and therefore the significance of the spectral peaks;
(2) to determine the Q factor of dissipation;
(3) to determine possible changes in time in the polarization of the individual modes.

A sampling interval of 1 min was adopted, since a simple inspection of the records shows that significant frequencies higher than 0.5 c/min do not occur.

The sampling was made on a photographic enlargement of the records magnifying the original hourly interval of 19 mm to 60 mm. The main oscillations for the shock as recorded at Trieste start at May 22-20 h 26 min, but the first 5 hours and 7 minutes on the EW and the first 2 hours and 53 minutes on the NS component are unusable because the speed of the light spot in the recorders was too high to leave a recognizable trace on the photographic paper.

3. For each 30-hour section, a drift described by a cubic parabola and the four main tidal waves $M_1$, $M_2$, $S_1$, and $S_2$ have been eliminated by least squares both to obtain an approximately stationary time series and to reduce the high energies contributed by tidal components and other very low frequency effects. As a consequence of such treatment, prewhitening procedures seemed to be unnecessary.

The lag in the correlation function was set at 1/6 of the length of each time series, i.e. at 300 min—a longer lag would have given a more detailed spectral resolution, but at the same time a coarser confidence interval, and thus a lower stability of the spectral peaks.

The auto- and cross-correlation functions were previously modulated with the $D_2(\tau)$ lag window, the $Q_2(f)$ spectral window of which eliminates the side lobes almost completely. Adjacent spectral peaks can thus be resolved if they are at least four elementary frequency bands apart.

In the power spectra (as well as in the cross-spectra), the elementary frequency band is $1.6 \times 10^{-2}$ c/min; consequently the periods of the lower modes $S_2$ and $T_2$ can be determined with an error of $\pm 2.4$ min and of $\pm 1.5$ min respectively (4-5% and 3-4%), while for the modes of order higher than the eighth, the error is reduced to less than 1% of the period.

The 95% confidence interval calculated from the $\chi^2$ distribution with 11 degrees of freedom, varies from 0.5 to 3 times the estimated value ($-5$ dB and 3 dB), the number of degrees of freedom being calculated from the Tukey formula $2[(T_m/T_n) - \frac{1}{2}]$ where $T_m$ is the length of the time series (1800 min) and $T_n$ is the lag of the auto-correlation function (300 min) (Blackmann & Tukey 1958).

It may be noted that since the modes with a period of less than four minutes have already disappeared in the second section, the spectra of the II, III, IV, and V sections have been calculated by using the alternate values of the auto-correlation function; the Nyquist frequency for these sections is thus 0-25 c/min.

Figs. 1 and 2 (where the ordinates are expressed in dB, the origin corresponding to the energy of $10^{-4}$ mm$^2$/c, the minimum level of the background noise) show the spectra of the five sections for each component separately.

In examining the spectra, the presence of aperiodic and very long period disturbances not completely eliminated in the preliminary treatment appears evident. The attraction and loading effects due to water movements in the upper Adriatic Sea and to the barometric pressure, which influence the horizontal pendulums, contribute to these disturbances (Caloi 1938, Polli 1958, 1960, Zadro 1962, 1964). In the course of time the loss of energy of the free oscillations of the Earth makes the asymptotic
Fig. 1. Power spectra of the EW component (original data).
Fig. 2. Power spectra of the NS component (original data).
Fig. 3. Power spectrum of the NS component, first section. A, original spectrum; B, the same corrected for the transfer function pertaining to deflections (radial modes); C, the same corrected for the transfer function pertaining to horizontal displacements (torsional modes).
behaviour of the background noise more and more evident; at the highest frequencies it approaches a white noise with energy levels of 15, 10, 3, 2, and 2 dB for the successive sections of the EW component and of 25, 5, 5, 0, and 10 dB for the NS component, respectively. The increase in the noise level in the last spectrum of the NS component may be explained by the worsening in the weather conditions causing fluctuations in the local barometric pressure which influences the NS component particularly (Zadro 1962).

By applying appropriate filters, a better spectral resolution may be obtained [this has been done (Zadro 1963) for the lowest frequencies]; nevertheless the original data have been used throughout in this analysis in order to obtain a first unbiased insight into the phenomenon.

The spectral values have not been reduced for the transfer function of the horizontal pendulums, since this function is different for the torsional modes (accelerations of the ground) and for the radial modes (deflections of the vertical) and the two modes overlap in the spectra and are not separable \textit{a priori}. In Fig. 3 the original spectrum (A) of the first section of the NS component is shown together with the spectra reduced according to the transfer function associated with the radial modes (B) and according to the transfer function associated with the torsional modes (C).

Finally we note that a judgement concerning the stability of the spectral peaks and therefore their physical reality should not rely only on the repetition of the spectral peaks in successive sections, since the appearance or disappearance of a peak may be influenced by the rotation of the polarization ellipse describing the relevant mode. The stability of the spectral peaks should be rather judged by the comparison between the spectra of the total energies associated with the individual modes, as discussed in the following paragraph.

4. Using the spectra of the two components and their cross spectra (for the calculation of which the length of the individual sections has been shortened by about two hours in order to assure simultaneity), the lengths of the axes of the polarization ellipses and their azimuths were calculated.

Fig. 4 depicts the five spectra of the major axes of the ellipses; for the more prominent peaks the ellipses are shown schematically themselves.

It may be observed that for the first two sections of 30 hours, and for the lowest frequencies where torsional and radial modes are clearly distinguishable, the torsional modes are polarized in the EW–SE direction and the radial modes in the NE–SW direction, as was foreseeable. Indeed the azimuth of the epicentre is in the NW direction at the Trieste Station.

In the first section, and in the frequency band between 0·10 and 0·17 c/min, the oscillations are strongly polarized in the NW–SE direction, although in each peak torsional and radial modes overlap; this fact apparently indicates a strong predominance of the torsional modes over the radial ones in the first hours of the record. The situation appears to change in favour of the radial modes in the second section.

In the following sections, the polarization for all oscillations does not follow a simple pattern, probably owing to the inhomogeneities of the Earth’s mantle and to local anisotropies of the crust.

The results thus reached allow the estimate of the total energy associated with each individual frequency band, as the simple sum of the energies associated with the two uncorrelated degrees of freedom of the movement; the spectra of the total energy are shown in Fig. 5.

By comparing the successive sections the stability of the spectral peaks appears now much more evident.

It may be noted that the modes of lowest order, up to the fourth, are absent or very weak in the first two sections, and become more evident in the third, and then die out in the last two sections.
Fig. 4. Spectra of the major axis of the polarization ellipse and distribution of the polarization ellipse versus frequency.
Fig. 5. Spectra of the total energy.
The $oS_2$ mode appears faintly in the second section only; it should however be noted that the spectral values of the lowest frequency bands are resolved rather poorly, and that a re-examination of those bands using a band-pass filter would be opportune. The $oT_2$ mode (shifted by one frequency band in the last section) and the $oS_3$ mode (which disappears in the last section only) are evident. The $oT_3$ mode appears weakly in the last section only. Almost all the higher order modes, until about the 10th order, are clearly recognizable; the $oT_6$, $oS_8$ and the $oS_9$ are completely or almost completely absent while the $oT_7$ and the $oS_7$ overlap. The energy attributed to the $oS_5$ mode could derive also from the $oS_6$ fundamental radial mode, since the two peaks overlap in the spectral analysis: a finer structure analysis would be necessary to separate them (Ness, Harrison & Slichter 1961).

In the still higher frequency bands, the torsional modes between the 11th and 21st orders inclusive and the radial modes of the immediately lower order overlap.

### Table 1

**Theoretical (Gutenberg–Bullen A model of the Earth) and observed periods (minutes)**

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The spectral peaks of the modes of order higher than the 21st which are evident in the first section, begin to die out already in the second section, and are badly recognizable in the successive ones.

It should be noted that because of the loss of energy in time, the spectral peaks of the highest frequencies die out gradually, so much that they are reduced to almost the noise level for periods shorter than 10 min in the last section.

The sequence of spectra of the total energies thus determined has permitted the calculation of the dissipation factor $Q$; the relevant study is published in the Rendiconti dell'Accademia Nazionale dei Lincei (Marussi 1966).

The spectra of the total energies have been used instead of the separate spectra, to assess the periods of the free oscillations of the Earth as recorded at the Trieste Station. Fig. 6 gives a visual summary of the observed periods for the five sections and of the theoretical values according to the Gutenberg–Bullen model A of the Earth (Pekeris, Alterman & Jarosh 1961, MacDonald & Ness 1961). The observed periods are shown by shaded rectangles (the more heavily shaded corresponding to the stronger peaks) with bases equal to the elementary frequency band, and centred on the resolved frequency value; the width of the rectangles thus corresponds to the inaccuracy of the observed value.

The numerical values for all five sections are listed in Table 1.

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References


