Strain determinations with tiltmeters in cavities

C. EBBLIN and M. ZADRO
Istituto di Geodesia e Geofisica, Trieste University, Trieste (Italy)
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ABSTRACT


A pair of tiltmeters installed in a cave, at the lower suspension of a pair of large horizontal pendulums, showed similar tilts related to tidal effects but different tilts connected to seiches in the nearby Gulf of Trieste or to underground river floods. Such a difference has been ascribed to the different strain characteristics caused by the phenomena: the former one being mainly a rotation and the latter chiefly a distortion. It is suggested that several tiltmeters installed in a cavity could supply data that would allow one to describe the non-dilatational part of the strain.

INTRODUCTION

Shortly before the destructive Friuli earthquake of 6 May 1976, the Istituto di Geodesia e Geofisica, University of Trieste, started installing in the seismic area a tiltmeter network to detect possible events preceding periods of stronger seismic activity (Zadro, 1978; Ebblin and Zadro, 1979a).

As a first stage of the project the behaviour of a pair of Marussi tiltmeters (Zadro, 1978) was investigated. The instruments were installed at the base of the pair of large horizontal pendulums operating in the cave of Grotta Gigante (Marussi, 1959; 1960; Zadro, 1962), and a comparison was made between the data obtained from the tiltmeters and those from the pendulums. The sensitivity of the tiltmeters was of 10 msec and that of the pendulums of 0.2 msec.

The tilts recorded by the two sets of instruments showed, among other things, the following peculiarities which are relevant to the purpose of the present paper.

(1) Tilts related (Zadro, 1966) to tidal phenomena were recorded by both kinds of instruments in the same direction with the same sense and intensity.

(2) Tilts ascribed (Zadro, 1964) to seiches (Fig. 1) in the nearby Gulf of Trieste or to floods of the underground Timavo river were recorded in different directions and with different intensities.
Fig. 1. Example of the two components of tilt recorded in the Grotta Gigante during a seiche in the nearby Gulf of Trieste. Full line: pendulum records; solid dots: tiltmeter records. The tiltmeter records have been enlarged approximately 3 times and 1 cm on the abscissa corresponds to 200 msec. The pendulum ones have been reduced to 1/3 of their original size and 1 cm on the abscissa corresponds to 25 msec. After 17h00 the tiltmeter data have not been recorded because of a temporary breakdown of the power supply.

The purpose of this paper is to give a possible explanation for these observations together with some suggestions regarding the potential use of tilt measurements in underground cavities.

ALTERATION OF DIRECTIONS AS A RESULT OF DEFORMATIONS

Let us consider the general case of a plane infinitesimal displacement gradient in two dimensions, the deformation being characterized by the matrix $E$:

$$E = \begin{bmatrix} \epsilon_1 & \omega \\ -\omega & \epsilon_2 \end{bmatrix}$$

where $\epsilon_1$ and $\epsilon_2$ are the principal extensions and $\omega$ the rotation. Any direction in the plane of the principal strains, characterized by an angular coefficient $k$ with respect to the fixed Cartesian reference axes chosen to be parallel to the principal directions, is deflected during the deformation $E$ to a direction with the coefficient:

$$k' = \frac{(1+\epsilon_2)k + \omega}{(1+\epsilon_1) - k\omega}$$

(1)

This result is valid independently of the particular choice of the reference system. The transformation of $k$ to $k'$ can be considered as the angular deformation of the corresponding affine transformation.

Neglecting higher-order infinitesimal terms and taking $|k| \leq 1$ ($|k| > 1$
would rotate the reference axes through more than 90\(^\circ\):

\[ k' = k + h(e_2 - e_1) + \omega(1 + k^2) \]  

(2)

The angle between the two directions is:

\[ \theta = \frac{k}{1 + k^2} (e_2 - e_1) + \omega \]  

(3)

which shows that the deflections are obviously independent of area changes, that in the case of rigid-body rotation or when \( e_1 = e_2 \), \( \theta = \omega \), and that \( \theta \) is equal to the angle of shear in the case of an irrotational strain. The two-dimensional case can be extended to three dimensions.

**TILT AND PENDULUM OBSERVATIONS IN CAVITIES**

In the following paragraph tilt and pendulum measurements are assumed to have been taken in a cavity deforming homogeneously with the continuum around it; in other words, special cavity effects are not taken into consideration. Moreover, for simplicity's sake, it is assumed that both the vertical direction and the poles to the tangent planes on the cavity walls on which the tiltmeters are mounted lie in one of the principal planes of strain; thus, if there is no strain perpendicular to this plane, i.e. plane strain, the conditions are those illustrated in the preceding paragraph.

As a consequence, a pendulum will record the deflection of the ideal line connecting its upper and lower mounting from the original vertical line, and this measurement will be independent of the position of the pendulum in the cavity. The deflection will only be a function of the constant angle between one of the principal axes of strain and the vertical. A tiltmeter, on the other hand, will record the deflections of an original line defined by the intersection of the local tangent plane to the cavity wall at the point where the instrument is installed, with the vertical principal plane of the strain (Ebbelin and Zadro, 1979b). The measurements of tiltmeters will therefore differ at different locations (Fig. 2).

It thus follows that pendulums will record no tilts in the case of irrotational strain with one principal direction vertical and that the only tiltmeters which will be unaffected are those mounted on a cavity wall the tangent plane of which is orthogonal to a principal direction.

The measurements taken in Grotta Gigante and the foregoing considerations suggest that, if both cavity effects on the strain fields and Newtonian effects due to the local redistribution of mass can be neglected, tidal phenomena will be recorded as rigid-body rotations of the crust, yielding the same measurements on pendulums and tiltmeters. On the other hand, different tilts will be recorded by the two sets of instruments as a consequence of seiches in the Gulf of Trieste or of floods of an underground river, showing that the deformations caused by these phenomena are at least partly distortional. Obviously, when a deformation is more
Fig. 2. Diagrammatic representation of the circular intersection between a vertical principal plane of a strain with a spherical cavity. The circle is deformed into an ellipse with the same area. The measured tilt varies from place to place whereas the pendulum measurements are independent of instrument position.

distortional than rotational, a tiltmeter may record a deflection with a sense opposite to that recorded by the pendulums mounted on the same base. In the simple case illustrated in Fig. 2, this happens if the instruments are located in the lowermost quadrant of the cavity.

In conclusion, we suggest that the non-dilational part of deformations of the Earth’s crust could be monitored using an array of tiltmeters installed in a cavity together with a pair of simple, low-cost, horizontal pendulums. The locations of the tiltmeters inside the cavity could be optimized for the geometry of any cavity, once the orientation of the expected principal strains is known.

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REFERENCES


