Variations of mechanical properties of rocks as a source of vertical ground displacements

C. EBBLIN, M. SALAHORIS and M. ZADRO

Istituto di Geodesia e Geofisica, University of Trieste, Via Università 7, Trieste (Italy)
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ABSTRACT


An analysis of the vertical ground displacements which might arise from modifications of the mechanical properties of rocks in the gravitational and a tectonic stress field shows that they are two orders of magnitude larger than those which can be related to comparable changes of density. It is also shown that a homogeneous tectonic stress field with the magnitude of the sum of its normal horizontal stresses lower than 10^9 dyn·cm^-2 generally yields negligible displacements compared to those in the gravitational field. Estimates of the changes in the mechanical properties and density of rocks involved in faulting from data on the variations of the velocities of seismic waves in the Friuli area show that the vertical displacements of tens of centimetres ascribable to such changes are of the same order of magnitude as those obtained from levelling surveys.

INTRODUCTION

Observations concerning the perturbations of the various geophysical fields in seismically active areas (cf. Rikitake, 1976) have led to the formulation of models based on laboratory fracturing experiments (cf. Brace et al., 1966; Scholz, 1968; etc.) and regarding the physics of earthquakes (Nur, 1972; Aggarwal et al., 1973; Whitcomb et al., 1973; Brady, 1974; etc.).

In most of these models the perturbations are explained in terms of the closing or opening of microfractures and of the flow of pore water from or into them observed in the laboratory when test samples are brought near failure conditions. Obviously such processes would result in modifications of, among other things, the density and the effective elastic parameters of the rocks involved in faulting.

In fact, variations of the velocities of seismic waves occurring in earthquake-prone areas in connection with periods of abnormally high seismic activity have been reported since the early 1960's (cf. Anderson and Whitcomb, 1975). Moreover, among other variations, those of the vertical displacement rates have also been observed since the early 1930's (cf. Rikitake, 1976) in the same conditions.
The purpose of the present paper is to discuss the effects of the changes of the mechanical properties and density of rocks involved in faulting processes on the vertical ground displacements.

VARIATIONS OF EFFECTIVE ELASTIC PARAMETERS

Although the physical modifications occurring in materials under near-failure conditions cannot be interpreted in terms of the theory of elasticity, at any single moment rocks in these conditions can be assumed to be characterized by a set of effective elastic parameters. However, since materials respond to stresses with certain elastic strains which are dependent on their elastic parameters, there will be strain differences ascribable to diverse elastic parameters in the same stress field. The vertical displacements connected with these particular strain differences are considered in this paper.

One way to estimate the effective elastic parameters of rocks in zones of seismic activity is from the velocities of seismic waves travelling in them according to the well-known formulae:

\[ \nu = \frac{V_p^2 - 2V_s^2}{2(V_p^2 - V_s^2)} \]

\[ \frac{E}{\rho} = \frac{V_s^2(3V_p^2 - 4V_s^2)}{V_p^2 - V_s^2} \]

where \( \nu \) is Poisson's ratio, \( E \) Young's modulus, \( \rho \) the density and \( V_p \) and \( V_s \) the velocities of longitudinal and transverse waves respectively.

Thus from the variations of the \( V_p \) and \( V_s \) recorded in the focal volume of an earthquake sequence, the variations of the effective Poisson's ratio of the rocks involved in the process might be computed together with the changes of the effective Young's modulus and density once a relationship between the variations of \( E \) and of \( \rho \) is assumed.

Thus, for example, from some reported data on the variations of the \( V_p \) and \( V_s \) in seismic areas (Scholz et al., 1973; Anderson and Whitcomb, 1975; Colautti et al., 1976) and assuming \( \Delta E/\Delta \rho \) of about \( 4 \times 10^{11} \) dyn cm g\(^{-1}\) (Youash, 1970), the corresponding maximum changes of \( E \) and \( \rho \) are of about 50% and of \( \nu \) of about 15% over several months and as much as 40% of \( \nu \) and 70% of \( E \), if \( \rho \) is taken to have varied not more than 50%, over several years.

VERTICAL DISPLACEMENTS

Considering a semi-infinite elastic homogeneous medium with a Cartesian reference system, its z-axis being vertical and positive downwards and the free surface at \( z = 0 \), according to the linear stress-strain equations for the static case, the vertical longitudinal strain is:
\[ \varepsilon_z = \frac{1}{E} \left[ -\nu (\sigma_x + \sigma_y) + \sigma_z + \frac{(1+\nu)(1-2\nu)}{1-\nu} \rho g z \right] \]

where \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are the normal stresses of a general homogeneous field, \( E, \nu, \rho \) are Young’s modulus, Poisson’s ratio and density, respectively, and \( g \) is the gravity (taken to be independent of depth).

The contribution to the vertical displacement at the surface \( w \) from a depth interval \( z, z+h \) can be found integrating the strain over such interval. The variations of that displacement \( \Delta w \) related to small variations of \( E, \nu, \rho \) can be evaluated considering the three corresponding differential quantities:

\[
\Delta w_E = -\frac{\nu (\sigma_x + \sigma_y) - \sigma_z}{E^2} h \Delta E + \frac{(1+\nu)(1-2\nu)}{2E^2 (1-\nu)} \rho g (h^2 + 2hz) \Delta E
\]

\[
\Delta w_\nu = \frac{\sigma_x + \sigma_y}{E} h \Delta \nu + \frac{\nu (2-\nu)}{E (1-\nu)^2} \rho g (h^2 + 2hz) \Delta \nu
\]

\[
\Delta w_\rho = -\frac{(1+\nu)(1-2\nu)}{2E (1-\nu)} g (h^2 + 2hz) \Delta \rho
\]

**DISCUSSION**

The length \( h \) over which the strain above was integrated represents, in the case of a seismic area, the vertical dimension of the volume of rocks in which the mechanical properties underwent a modification, and \( z \) the depth below which the volume is situated, both of them vertically under the point in consideration on the surface. Obviously for a particular fault size at a given depth the dimension \( h \) depends on the fault dip and is therefore related to the orientation of the principal stresses, largest ground displacements being expected when the direction of the intermediate principal stress lies in a vertical plane.

From the relationships above it can be seen that the effects of changes of Young’s modulus in the tectonic stress field and in the gravitational field counteract. In fact, the term involving the tectonic stresses displays a sign opposite to that where the gravity appears. On the contrary modifications of Poisson’s ratio produce the same effect both in the tectonic and gravitational fields.

In cases of increase of Young’s modulus the ground over the rocks involved in the change will subside when the effect of the tectonic stress field will be predominant whereas uplift will occur when the gravity effect will prevail. The dashed curves in Fig. 1 represent the loci of points of the \( \nu h \) plane for which modifications of \( E \) yield no vertical displacements. Hence for any pair of \( \nu, h \) values above or to the right of these curves subsidence will occur while for any pair of \( \nu, h \) values below or to the left of the same curves uplift will be observed. It might be noticed that these curves in the
Fig. 1. Curves representing the loci of points $\nu$, $h$ for which variations of Young's modulus yield no vertical displacements (broken lines) and variations of Poisson's ratio yield the same displacements in the tectonic and gravitational fields (full lines) according to the elastic theory. $\sigma = \sigma_x + \sigma_y$ in $10^8$ dyn cm$^{-2}$ = 10$^5$ bar = 10 MPa, $\sigma_z = 0$, $\rho = 2.67$ g cm$^{-3}$ and $g = 981$ cm s$^{-2}$.

The figure represents the conditions for different depths but for the same value of $\sigma_x + \sigma_y$. Such value was taken to be $5 \cdot 10^8$ dyn cm$^{-2}$, almost twice the maximum value found from field measurements (Hast, 1973; Rummel, 1979); however, at lower tectonic stresses the curves were found even further to the right showing that subsidence could occur only in case of almost incompressible rocks. Thus, subsidence is to be generally expected when the horizontal tectonic stresses are very high, the rocks are characterized by high Poisson's ratio and the volume of rocks involved in the change is small and at shallow depth.

In cases of increase of Poisson's ratio instead, both the gravity and tectonic fields produce uplift and thus the full curves in Fig. 1 represent the loci of points of the $\nu h$ plane for which the contribution of the two fields is the same. Obviously in a volume of rocks characterized by a vertical dimension $h$ and a Poisson's ratio $\nu$, which define a point above or to the left of the curves, the contribution, in terms of vertical displacements, of changes of $\nu$ in the tectonic field will be predominant, while if the point is below or to the right, the effect of the gravity field will prevail. Even here it is seen that the depth $z$ is important in determining the relative contribution of the tectonic and gravity field and that it is counterbalanced by the intensity of the tectonic field. In fact an increase of $z$ shifts these curves to the left while and increase of $\sigma_x + \sigma_y$ displaces them to the right.

Comparing now the vertical displacements related to variations of $E$, to those connected with changes of $\nu$, and to those due to modifications of $\rho$ (Fig. 2) it is apparent that for the same percentual changes, in the same
Fig. 2. Vertical displacements as a function of modifications of Young’s modulus (broken lines), of Poisson’s ratio (full lines) and of density (dotted line) according to the elastic theory. \( \sigma = \sigma_x + \sigma_y \) in 10^9 dyn cm\(^{-2} \), \( \sigma_z = 0 \), \( z = 5 \) km, \( h = 1 \) km, \( E = 10^{11} \) dyn cm\(^{-2} \), \( \nu = 0.25 \), \( \rho = 2.67 \) g cm\(^{-3} \) and \( g = 981 \) cm s\(^{-2} \).

conditions, the displacements related to \( \Delta \rho \) will be about two orders of magnitude smaller than those connected with \( \Delta \nu \). Obviously since the displacements related to \( \Delta E \) are, in the tectonic and gravitational fields, of opposite sign \( \Delta w_E \) will instead depend on the magnitude of \( \sigma_x + \sigma_y \). For values of \( \sigma_x + \sigma_y \) of the order of 10^8 dyn cm\(^{-2} \) the effect of the gravity field will be largely predominant and \( \Delta w_E \) will be of the same order of magnitude as \( \Delta w_\nu \). The magnitude of \( \Delta w_E \) will drop instead for values of \( \sigma_x + \sigma_y \) higher than 10^9 dyn cm\(^{-2} \) and will finally increase again, after changing sign, at even higher tectonic stresses.

However, since the field observations mentioned above not only suggest that \( \sigma_x + \sigma_y \) does not reach values of 10^9 dyn cm\(^{-2} \) but also that \( \Delta \nu/\nu \) is generally half of \( \Delta E/E \) it must be concluded that the latter changes are those that most strongly influence the vertical displacements.

THE FRIULI EXAMPLE

Levelling observations carried out in the seismic area of northern Friuli, Italy, have revealed a maximum difference of vertical displacements of 24 cm over a distance of 5 km in the epicentral zone in the last 25 years (Talamo et
al., 1978). However, the pattern of such displacements is hardly explainable in terms of dislocations connected with slip on a fault characterized by the focal parameters obtained from seismological data (Müller, 1977; Cipar, 1980; Ebblin, 1980; Lyon-Caen, 1980; Stoll, 1980).

Tests carried out (Salahoris, 1981) on a two-dimensional finite-elements model of a N–S cross-section of Friuli with the mountains on one side of the epicentral section and the alluvial plain on the other have shown that the introduction of a proper area where the elastic parameters and the density have changed according to the data on the $V_p$ and $V_s$ observed immediately after the May 6, 1976 destructive shock, yields vertical ground displacements comparable to those obtained from the levelling observations.

Thus, it is suggested that vertical ground displacements in seismic areas related to modifications of the compressibility of rocks involved in faulting processes are generally more significant than those connected with variations of the density owing to the opening or closing of microfractures, and those of the shear modulus.

CONCLUSIONS

The analysis above suggests that modifications in time of the mechanical properties of rocks cause vertical displacements which are two orders of magnitude larger than those which might be ascribed to variations of their density. Moreover, it is seen that in cases where the tectonic stress field is of the order of magnitude like that obtained from in-situ measurements (Hast, 1973; Rummel, 1979) the displacements connected with the gravitational field are generally preponderant.

Similar conclusions might be expected if viscous media and/or if spatial variations of the mechanical properties were instead taken into consideration. Hence it is suggested that models constructed for the estimation of the vertical displacements include as variables the mechanical properties of the rocks.

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REFERENCES


