MULTIPLE-INPUT LINEAR SYSTEMS IN GEOPROCESSES: AN ANALYSIS OF GEOPHYSICAL DATA ACROSS THE EASTERN CONTINENTAL HELLENIC MARGIN

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Abstract

Statistical methods introduced and developed in applied sciences by researchers in informatics and electronic engineering, have been applied until now to geoscience problems mostly for single processes (spectral techniques) or single input-output linear system processes (filtering-deconvolution and deconvolution techniques).

In the present paper a more wide use of the available statistical techniques is proposed, based on the Multiple Linear Input System Theory, in order to study the mutual dependence of several geophysical processes which may or may not be correlated with one another.

As an example, a profile through a geophysically interesting area of the Aegean basin is considered and the coherence and gain factors of several geo-signals like geoidal heights, Bouguer anomalies, depth to the Moho, bathymetry, and heat flow are computed. Suggestions for further investigations are given, especially for a two-dimensional approach.

1. INTRODUCTION

Statistical methods applied to the treatment of time series, were initially developed and widely used in electric and electronic engineering. In the last decades, however, they also found various applications in different branches of Earth sciences. Until now statistical methods have been successfully used in geosciences to analyse single processes (power spectrum analysis) or to compare two sets of data, e.g. topography and gravity anomaly profiles, tides and atmospheric pressure, etc., mainly using filtering techniques, auto- and cross-correlation functions and their spectra and coherence analyses (e.g. Lee and MacDonald, 1963; Munk and Cartwright, 1966; Kanasewich and Agarwal, 1970;
McKenzie and Bowin, 1976; Vyskočil and Burda, 1978). This approach, which in terms of signal theory correspond to a single input-output process, may lead to misleading results whenever the two geoprocesses under comparison are dependent on some other process which has not been taken under consideration. It is the scope of this paper to illustrate how the Multiple Input Linear System (MILS) technique, widely used by electronic engineers in information theory problems, may be usefully applied to geosciences when studying interrelations between various geodetic and/or geophysical fields. The MILS procedure overcomes indeed the above-mentioned limitation by allowing the involvement of many correlated or uncorrelated geoprocesses in a unique system.

The data similar to those analyzed here are in practice not so sensible to high frequency noise as the data analyzed in other applications when incomplete unequally spaced data are used or short wavelength noise may disturb considerably the input and output data, in a way that other techniques can be alternatively tried (see, e.g., Vaniček, 1971; Merry and Vaniček, 1981; Steeves, 1981; Delikaraoglou, 1983).

In order to apply the MILS technique, the stationarity of stochastic processes has to be assumed as well as the linearity of a given system (like in a single input-output case). Although these conditions are not strictly satisfied by geoprocesses usually under consideration, the method allows to obtain a first order approximation solution. The example described in this paper can be considered as a representative one.

A profile in the SE Aegean area of about 200 km long was analyzed by considering as mutually dependent signals the bathymetry, the Bouguer anomalies, the Moho depth, the heat flow, and the geoidal heights. The scope of the analysis was the study of physical consistency of such a system as well as the determination of the signal which gives the best linear dependence upon the others ones. This study, being a first approach in treating similar problems in geosciences, was carried out in one dimension. Extensions to two-dimensional fields is also suggested for better understanding in a more complete way involved of the geoprocesses.

2. MULTIPLE INPUT LINEAR SYSTEM (MILS)

A MILS can be illustrated as in Fig. 1a, where \( x_1, x_2, \ldots, x_n \) are given input signals of \( n \) corresponding linear filters working in parallel, the unknown outputs of which \( y_1, y_2, \ldots, y_n \), are added to a known unique output \( y \). \( h_1, h_2, \ldots, h_n \) are the impulse response functions of the filters and \( H_1, H_2, \ldots, H_n \) are their Fourier transforms, i.e. the frequency response functions. \( H_1, H_2, \ldots, H_n \) are the corresponding gain factors of the MILS. The input signals may or may not be correlated.

It should be noted that an \( H_i \) filter can represent a set of \( m \) linear filters working in series, thus giving directly the linear transformation which ties the first input \( x_i \) with the last output \( y_i \) of the set (Fig. 1b). In that case the frequency response function \( H_i \) is the product of the \( m \) corresponding frequency response functions.

Couples of inputs strongly correlated with each other have to be avoided since one of them can be considered as the output of a filter having the second one as the input (Fig. 1c),
so that the same input results as acting through two different filters thus hindering the solution of the system.

The frequency response functions $H_i$ are linearly related to the auto- and cross-spectra of the input-output signals as follows:

$$g_{xi} = \sum_{j=1}^{n} H_j g_{xi,j}, \quad i = 1, n.$$  \hspace{0.5cm} (1)

The above system is forming, frequency by frequency, an $n \times n$ linear system, $g_{xi}$ and $g_{xi,j}$ being the complex spectra mentioned above. Of particular interest, in such a type of problems, is the partial coherence $\gamma_{xi}^2$ defined as

$$\gamma_{xi}^2 = \frac{|g_{xi}|^2}{g_{xi,xi} g_{yy}}, \quad 0 \leq \gamma_{xi}^2 \leq 1,$$  \hspace{0.5cm} (2)

where $g$ are the residual spectra (Bendat and Piersol, 1971) which can be computed from the above MILS by substracting the energy contribution coming from the correlated quantities between the input $x_i$ (or output $y$) and all the other inputs. It is interesting to note that the partial coherence (2) remains invariant when interchanging the input signals $x_i$ with the output $y$ ($\gamma_{yi}^2 = \gamma_{xy}^2$).
A second kind of coherence which is of particular interest is the multiple coherence $\Gamma^2$ between the output $y$ and all the inputs $x_i$, which is defined as (Bendat and Piersol, 1980):

$$\Gamma^2 = 1 - \frac{\theta_{yn}}{\theta_{yy}},$$

(3)

where $\theta_{yn}$ is the extraneous noise output spectrum. Thus the multiple coherence $\Gamma^2$ represents a measure of the linear dependence of all the $x_i$ upon the $y$ signal as well as of the physical consistency of the whole linear system which has been considered.

In treating MILS problem numerically, rounding-off errors and inappropriate estimates of spectra ought to be carefully avoided while computing the coherences and the gain factors. For this reason the input and the output signals have to be reduced to zero mean values and to comparable order of magnitude. Application of a suitable lag window to the correlation functions or direct smoothing of the spectra with the corresponding frequency filter avoid jumps in the raw spectra, which usually cause inappropriate estimates of the coherence. The D2 lag window (Blackman and Tukey, 1958), which is generally used for smoothing the raw spectra, is not appropriate in many applications and a more efficient smoothing by applying the Parzen lag window (Kanasewich, 1981) is advisable.

3. APPLICATION OF THE MILS TECHNIQUE TO THE EASTERN HELLENIC MARGIN

A profile passing through one of the most tectonically unstable areas of Europe, the Aegean basin, was considered. This profile (Fig. 2) extends for about 200 km across the eastern continental Hellenic margin, at the contact zone between the Indian-Arabian and the Euro-Asian plates. It is almost perpendicular to the direction of the eastern part of the Hellenic Arc and it crosses the South Aegean Volcanic Arc as well as the line which extends from Creta to the North of Rhodes (Angelier et al., 1982; De Bremaecker et al., 1982) where the $\sigma_2 - \sigma_3$ stress trajectories ($\sigma_1$ assumed to be vertical) interchange and the strain trajectories show a similar interchanging of the intermediate with the greatest shortening strain. North of this line the Aegean basin undergoes a principal extension in an about N-S direction, while south of it the extension is in a roughly NE-SW direction.

The geophysical signals used for this profile (Fig. 3) are the bathymetry ($T$), the complete Bouguer anomalies ($B$) (Morelli et al., 1975), the Moho depths ($M$) (Makris, 1977), the heat flow ($F$) (Fytikas and Koliros, 1979), and the geoidal heights ($G$) (Cruz and Rapp, 1982) referred to the GRS system computed from SEASAT altimetric data. All the signals show a steep gradient, especially when approaching the Hellenic Arc zone. The signals are sampled at every 2.5 km. The profile as well as the sampling have been constrained by the availability, density and homogeneity of the data. The accuracy of the data read off the maps is not, unfortunately, as high as that required by the method and a consistent level of noise has to be assumed in the signals.

A lag of 10% was adopted for the correlation functions in all the MILS tests carried out, and nine frequency bands were solved between 0.0 and 0.2 cycles km$^{-1}$ for the spec-
tral entities as well as for the coherences and gain factors. The first one (infinite wavelength) and the last two (6.1 and 5.0 km of wavelength) were disregarded as having no meaning and scarce reliability.

Several tests were run using a specially written computer program and adopting, in turn, one or the other of the above-mentioned signals as the output, the other ones being the inputs. The reliability and physical consistency of the considered systems was judged on the basis of coherence functions.

The above tests are suitable whenever the physical meaning of the interrelations occurring among the signals is not "a priori" known (like in the present example the relations concerning the heat flow). Moreover, following such a procedure it is possible to avoid cases like the one illustrated in Fig. 1c, just considering one of the two highly partially coherent signals as output and the other one together with all the remaining others as inputs. The selection between the two possible MILS obtained by interchanging the input-output role should be made on the basis of the multiple coherence function, selecting the system with its highest values.

The heat flow signal appeared to be the least consistent signal. Moreover, the same tests carried out without taking into account the heat flow as an input signal showed general improvement in the reliability of the results. For these reasons the heat flow was disregarded even as an input signal, except for some test referred further on.
Fig. 3. Five geophysical profiles along section AB

T - bathymetry, M - Moho depths, F - heat flow,
B - Bouguer anomalies, G - geoidal heights
As shown in Fig. 4, among the various MILS tested, MILS-G (geoidal heights as the output) appears to be the most consistent one, followed by MILS-M (Moho depth as the output). The multiple coherences in the two cases are indeed almost coincident, except for the wave lengths centred at about 10 km where the both cases present their minimum values ($\Gamma_G^2 = 0.9$ and $\Gamma_M^2 = 0.7$). The multiple coherences of MILS-B (Bouguer anomaly as the output) and particularly of MILS-T (bathymetry profile as the output) indicate generally lower consistency of the corresponding systems. For wavelengths at 10 and 9 km they show unacceptable values (beyond the 0 - 1 range) and hence are not represented in the figure.

![Figure 4](image-url)  
**Fig. 4.** Multiple coherences over the resolved wavelengths of MILS-Geoid ($\Gamma_G^2$), MILS-Moho ($\Gamma_M^2$), MILS-Bathymetry ($\Gamma_T^2$), and MILS-Bouguer ($\Gamma_B^2$)

![Figure 5](image-url)  
**Fig. 5.** Partial coherences over the resolved wavelengths of $\gamma_{GM}^2$ (Geoid-Moho), $\gamma_{MB}^2$ (Moho-Bouguer), $\gamma_{GB}^2$ (Geoid-Bouguer), $\gamma_{GT}^2$ (Geoid-Bathymetry), $\gamma_{TM}^2$ (Bathymetry-Moho)

The most interesting partial coherences are shown in Fig. 5. The $\gamma_{GM}^2$ relating the geoidal to the Moho profiles are stable and highly coherent ($\gamma_{GM}^2 = 0.9$ everywhere). The coherences $\gamma_{MB}^2$ referred to the Moho depth and Bouguer anomalies, as well as $\gamma_{GB}^2$ referred to the geoidal heights and Bouguer anomalies present high coherences varying from 0.7 to 0.9. All partial coherences referred to the bathymetry profile are varying at considerably lower levels.

From the above observations on the partial and multiple coherence criterion for the selection of the most reliable MILS it results that the MILS-G is the best system. Indeed,
since the geoidal height is highly correlated with both the Moho and Bouguer anomalies profiles, its position as the output overcomes difficulties in the solution of the system. Still some problems may derive from the couple of signals \( M \) and \( B \) which have to act as inputs and appear highly partially mutually coherent. Nevertheless, both on the base of the relatively lower values of \( \gamma_{KM}^2 \) with respect to \( \gamma_{GM}^2 \) and \( \gamma_{GB}^2 \) and of the multiple coherence results, the selection of MILS-G turns out to be the most suitable.

The gain factors of MILS-G wavelength functions are listed in Table 1. Taking into account the partial coherence values, it appears that long wavelengths (about 40 km) and wavelengths close to 9 km are significant in terms of the bathymetric signal, where 100 m of depth variation causes about 1 m variation in the geoidal height. From the gain factor \( h_{GB} \) which associates the Bouguer anomaly as the input and the geoidal heights as the output, \( \gamma_{GB}^2 \) being rather high along the whole range of wavelengths, it results that 10 mGal variation corresponds to 1 - 2 m of geoidal height variation. For the gain factor \( h_{GM} \), which relates the input signal \( M \) (Moho depths) to the output \( G \) (geoidal heights), it results that 1 km of Moho variation corresponds to about 1 m of geoidal height variation. Checking the gain factors \( h_{TG} \), \( h_{BG} \) and \( h_{MG} \) computed from the MILS-T, MILS-B and MILS-M solution, respectively, we found a good correspondence only between \( h_{BG} \) and \( h_{GB} \) and between \( h_{MG} \) and \( h_{GM} \). The gain factor \( h_{TG} \) instead, computed from MILS-T, does not agree with the factor \( h_{GT} \) computed from MILS-G. In fact it reflects the obvious consequence of the ill-conditioning of MILS-T, where all three \((M), (B), (G)\) signals, which

\[
\begin{array}{c|c|c|c}
\text{Wave length} & h_{GT} & h_{GB} & h_{GM} \\
[km] & & & \\
42.5 & 1.1 & 0.2 & 1.2 \\
21.3 & 1.0 & 0.2 & 1.2 \\
14.1 & 0.5 & 0.2 & 1.2 \\
10.1 & 0.7 & 0.1 & 1.0 \\
8.5 & 1.4 & 0.1 & 0.9 \\
7.1 & 1.1 & 0.2 & 1.0 \\
\end{array}
\]

Table 1

The gain factors of MILS-G (the output: geoid). Solution without heat flow among the inputs

\[
\begin{array}{c|c|c|c}
\text{Wave length} & h_{GT} & h_{GB} & h_{GM} \\
[km] & & & \\
42.5 & 0.7 & 0.1 & 1.3 \\
21.3 & 0.6 & 0.1 & 1.3 \\
14.1 & 0.3 & 0.1 & 1.4 \\
10.1 & 0.7 & 0.1 & 1.1 \\
8.5 & 1.2 & 0.2 & 0.8 \\
7.1 & 0.9 & 0.0 & 1.2 \\
\end{array}
\]

Table 2

The gain factors of MILS-G (the output: geoid). Solution with heat flow among the inputs
are highly correlated between each other, appear as inputs, whereas the output is poorly correlated with the inputs.

As far as the phase lags between each input and the output (G) are concerned, the computations gave almost in-phase effects at every wavelength. It means that an elevation of the geoidal profile corresponds to an elevation of the Moho, topography and Bouguer anomaly profiles.

The tests carried out with heat flow as the signal provided, as mentioned above, reliable results according to the multiple coherence criterium only for MILS-G. Nevertheless, the partial coherences presented general deterioration of $\gamma_{GB}^2$ ($\gamma_{GB}^2 \leq 0.2$), while $\gamma_{GM}^2$ remained almost constant ($\gamma_{GM}^2 = 0.9$).

4. CONCLUDING REMARKS

The geophysical case considered in this paper is a representative example of interrelated signals, the signals being the bathymetry, the Moho depths, the heat flow, the Bouguer anomalies and the geoidal heights along a profile at the south-eastern continental margin of the Aegean basin.

The best system, judging on the basis of the coherence analysis, is MILS-G. It means that the best linear relationship which can be established among the considered signals is the one which corresponds to a system having geoidal heights as the output, the other signals being the inputs. From the different inputs of the MILS-G system the heat flow appears to be the least at the considered wavelengths and for this particular case.

The system which from the statistical point of view is the most reliable one, is also the same, which seems the most reasonable from the physical point of view. This can be easily argued from the following considerations.

With the exception of the heat flow signal, linear relationships connecting other signals can be found. In fact, geoidal undulations can be computed from gravity anomalies through a convolution (T'suboi, 1983). The gravitational field produced by a perturbing underground layer can also be considered as a result of the convolution between the mass distribution and weighing function depending on the distance alone. For the cause-effect principle, direct filters are those which consider the geoidal heights as an output and the other signals as inputs.

High-level partial coherences were found between the couples of signals like geoidal heights, Moho depths and Bouguer anomalies ($\gamma_{GM}^2$, $\gamma_{GB}^2$, $\gamma_{MB}^2$), whereas signals like bathymetry and geoidal heights ($\gamma_{GT}^2$) and especially bathymetry and Moho depths ($\gamma_{TM}^2$) are almost incoherent. The latter two cases can be interpreted, at least at the rather short wavelengths considered here, as a reasonable consequence of recent and uncompensated geodynamical processes.

The gain factors, according to the MILS technique, give the spectral output for a unitary amplitude input, having cancelled the possible contributions from all the other considered inputs both in the output and in the input itself. Thus it results that over all the considered wavelengths the gain factors computed for the bathymetry, the Moho depths, the Bouguer anomalies as the signals appear to be at least one order of magnitude greater than
the values reasonably expected, taking into account the mean values of the bottom and Moho profiles as well as the involved wavelengths. For the bathymetry signal, the low degree of coherence with the output allows to consider that results as scarcely reliable. Nevertheless, the large response due to the other two inputs remains still surprising. Although uncertainties in details of the input data could have given noisy signals responsible for the deterioration of the results, the high level of coherence suggests other hypotheses. It seems that large lateral and vertical variations of densities, which can reasonably occur in the zone, could be responsible for large variations both in the geoidal heights and in the Bouguer anomalies.

It should be noted that the axis of bending structures of the underlying layers rotates as a consequence of the interchange between the intermediate and the greatest shortening strain in the zone. This hypothesis is strengthened by the U-shaped pattern of all the considered fields. For that reason the same type of geological structures, with the same wavelength in the bidimensional domain, appear with different wavelengths on the profile when passing from the northern to the southern part of it.

The first consequence of the above-mentioned sudden rotation of the axis of underlying structures is that the gain factor does not change significantly, as it should, with the wavelength. The second consequence is the contribution to the output of lateral variations of densities ranging in the considered wavelength band and aligned with the profile itself along a consistent part of it. The above question connected with the lateral variations of densities could be solved by using the MFLS techniques in the two-dimensional domain.

However, although the density variations occurring laterally to the profile could have given some extra contribution to the gain factors, the output response still appears as too high. Thus apart from lateral variations, indeed, large vertical variations of densities are to be expected, at least in the considered wavelength band. These might be related to the presence of the crystalline layer which reaches considerable thickness in the Aegean basin and which at short wavelengths may follow the trend of the Moho depth.

Moreover, the basic magma chambers are present (Innocenti et al., 1981) at shallow levels in the eastern sector of the Aegean volcanic arc, highly probably corresponding to the maxima of Moho undulations, thus producing large gravitational effects at short wavelengths. The bathymetry, Moho depth and heat flow measurements presumably could not detect them in detail, as the corresponding perturbations are highly local and limited in spatial extent.

Due to the lack of detailed information it has not been possible to take into account other signals such as lithospheric and crystalline depths (especially for the involved wavelengths). Moreover, the details, at least in the heat flow data, seem to be questionable.

Although the geophysical example considered in this paper is a highly complex one, both as a result of the lack of sufficiently detailed information in the short profile considered and of geodynamical complexity of the zone involved, from the methodological point of view the following conclusions can be drawn.

The first conclusion is related to the power of the method which allows to extract, from a variety of geosignals, those which are interacting and to find the best linear relationship among them, provided that the data with a high signal-noise ratio are available. The second conclusion is that the dependence between the couples of signals has to be analyzed,
taking into account simultaneously all possibly correlated signals by using several tests and by neglecting afterwards extraneous signals on the basis of the corresponding results. The third conclusion is the advisability of applying the MILS technique in two-dimensional domains.

ACKNOWLEDGMENTS

Dr. C. Ebblin read critically the text and gave useful suggestions.

Manuscript received 23 November 1984

REFERENCES


WIELOWEJŚCIOWE SYSTEMY LINIOWE W PROCESACH GEOFIZYCZNYCH:
ANALIZA DANYCH GEOFIZYCZNYCH
W POPRZECIE WŚRODNOHELLEŃSKIEJ KRAWĘDZI KONTYNENTALNEJ

Streszczenie

Metody statystyczne, wprowadzone i rozwijane w naukach stosowanych przez informatyków i elektrowników, w naukach o Ziemi stosowano dotychczas w ograniczonym zakresie — przede wszystkim do badania pojedynczych procesów (techniki spektralne) lub w procesach opisywanych systemem liniowym z pojedynczym wejściem i wyjściem (techniki filtracyjno-konwolucyjne i dekonwolucyjne). W pracy proponuje się znacznie rozszerzone zastosowanie dostępnych metod statystycznych, których podstawę stanowi teoria wielowęściowych układów liniowych. Pozwolą one badać wzajemne zależności wielu procesów geofizycznych, które mogą korelować lub nie korelować między sobą.