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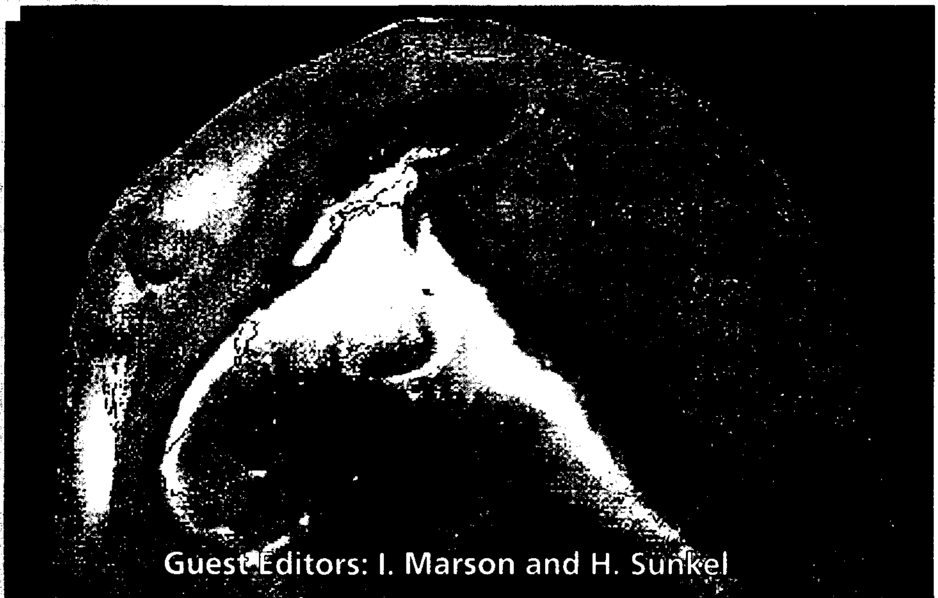

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Iterative 3D gravity inversion with integration of seismologic data

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Abstract. Depending on the a priori knowledge, different methods can be chosen for gravity inversion. The method we propose is suitable for cases in which little is known about the crustal structure, except the presence of a sharp density contrast, as occurs at the Moho or at the base of a sedimentary basin. The method combines the downward continuation with the direct evaluation of the gravity field of the undulations of the boundary surface in an iterative process. The a priori assumption of the reference depth of the boundary and density across the surface is necessary. If the depth of the boundary is known along a profile or in single points from seismologic (or other) investigations, this piece of information is used to anchor the boundary. In lack of such knowledge general geophysical considerations must be used to obtain an estimate of the boundary in the undisturbed state. The method is tested in a 3D synthetic model situation, evaluating the effect of erroneous a priori assumptions and noise.

1. The inversion method

We consider the problem of determining the oscillations of an horizontal boundary, which is part of a layered structure model of the lithosphere. This boundary could be for example the crust/mantle interface or the base of a sedimentary basin. This problem has been approached before by Parker (1972) and Granser (1987). The methodology we propose is an iterative solution that alternates the downward continuation with the direct calculation of the gravity field of the model. The method has been tested in 2.5D synthetic model situations (Braitenberg et al., 1997), and is extended to 3D synthetic models here.

Let $g_0\ (x, y)$ be the Bouguer gravity anomaly field, and d be the reference depth of the boundary. Let $r\ (x, y)$ be the oscillation of the boundary, defined as the deviation from the depth

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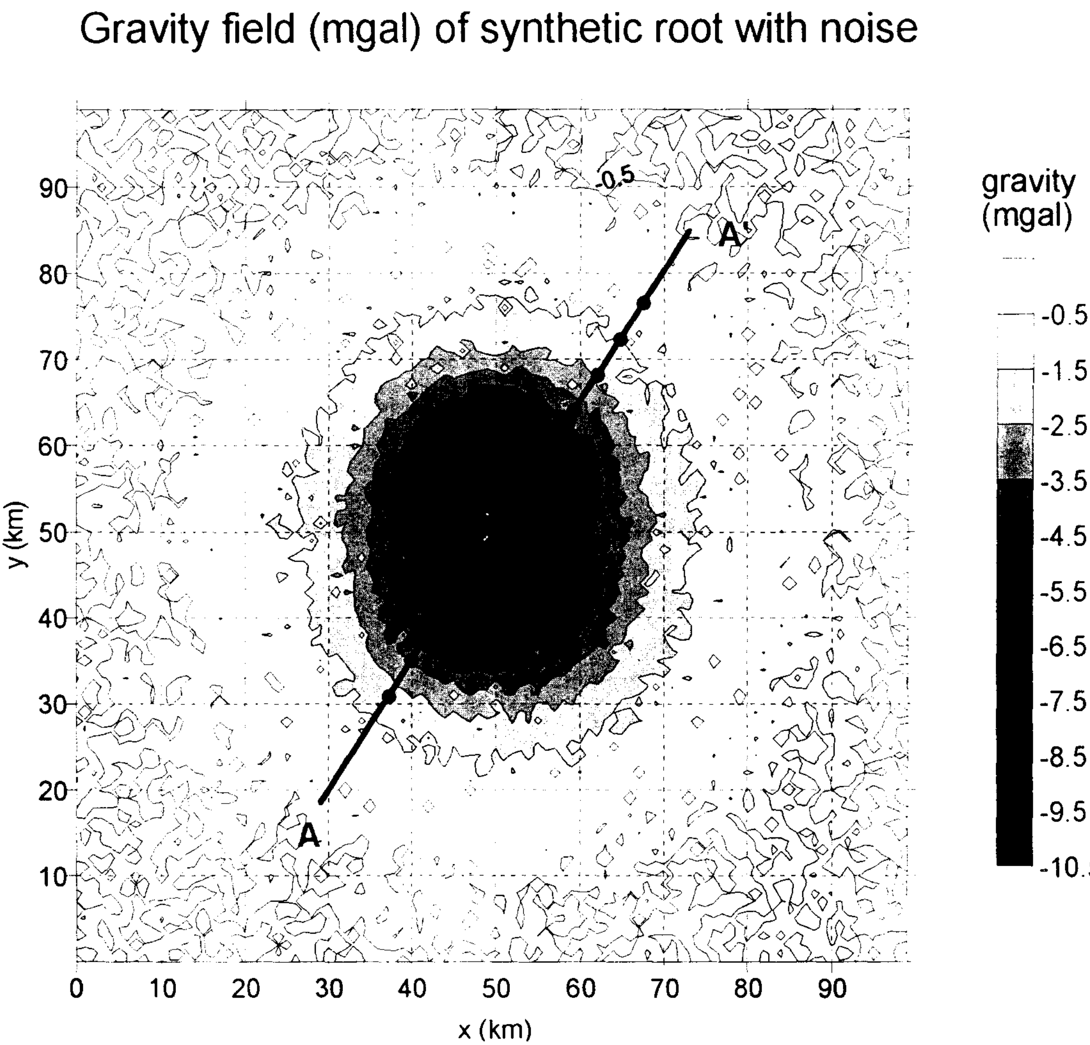

Fig. 1 - Synthetic gravity field with gaussian noise (0.5 mgal standard deviation). Model: two superposed prisms. Extension of the prisms is 40-60km, 35-65km, 8-9km upper, 45-55km, 45-55km, 9-10km lower, respectively. Density –0.4 g/cm^3. The dots refer to the position of constraining depths. Profile AA' is discussed in the following.

d. With $g_d\ (x,\ y)$ the downward continued field to the depth *d*, and on taking the Fourier Transforms (FT) of the field, we obtain:

$$g_d(k_x, k_y) = e^{d\gamma} g_o(k_x, k_y) \qquad (1)$$

$$\gamma = \sqrt{k_x^2 + k_y^2}$$

where k_x, k_y are the wavenumbers along the coordinate axes.

Assuming that the field is generated by a sheet mass located at depth *d*, the surface density of the sheet mass $\sigma\ (x,\ y)$ is given by:

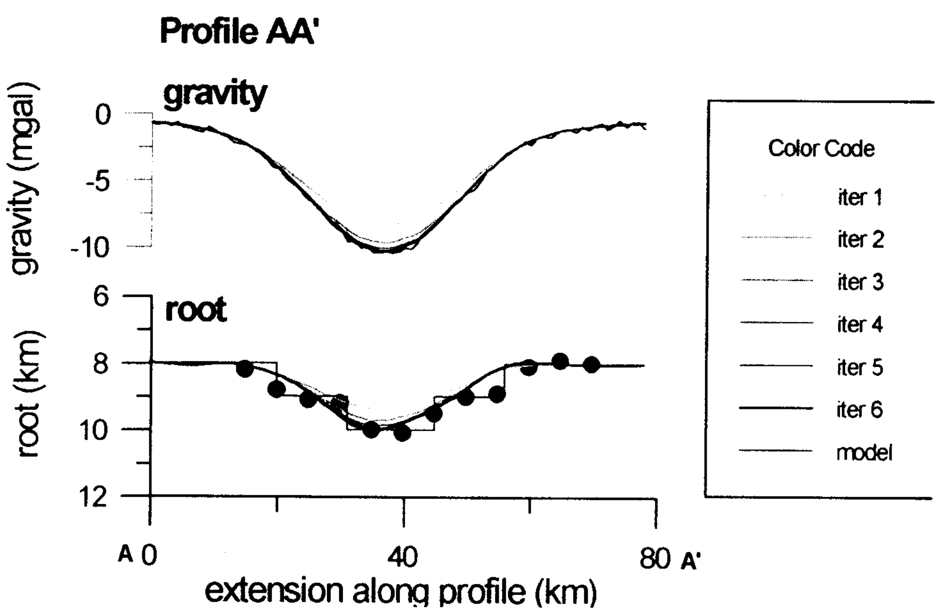

Fig. 2 - Model gravity and model root along profile AA' and respective inverted curves for iteration steps 1 through 6.

$$\sigma(x,y)=\frac{1}{2\pi G}g_d(x,y)=\frac{1}{2\pi G}FT^{-1}\left[g_d(k_x,k_y)\right], \tag{2}$$

with FT^{-1} the inverse Fourier Transform and G the gravitational constant. We may interpret the sheet mass with horizontally varying surface density as the oscillating boundary which separates two layers, and across which we have a density contrast $\Delta\rho$. The oscillation amplitude of the boundary is then given by

$$r_1(x,y)=\frac{1}{\Delta\rho}\sigma(x,y). \tag{3}$$

Only in first approximation though does the gravity field generated by the vertically expanded boundary coincide with the field $g_0\,(x,\,y)$, as shown by Parker (1977). We approximate the vertically expanded boundary with a series of rectangular prisms and calculate the gravity field by applying the algorithm developed by Nagy (1966). The residual gravity field $\delta g_1\,(x,\,y)$ is defined as the difference between the observed field ($g_0\,(x,y)$) and the field ($g_1\,(x,y)$) generated by the series of prisms,

$$\delta g_1(x,y)=g_0(x,y)-g_1(x,y). \tag{4}$$

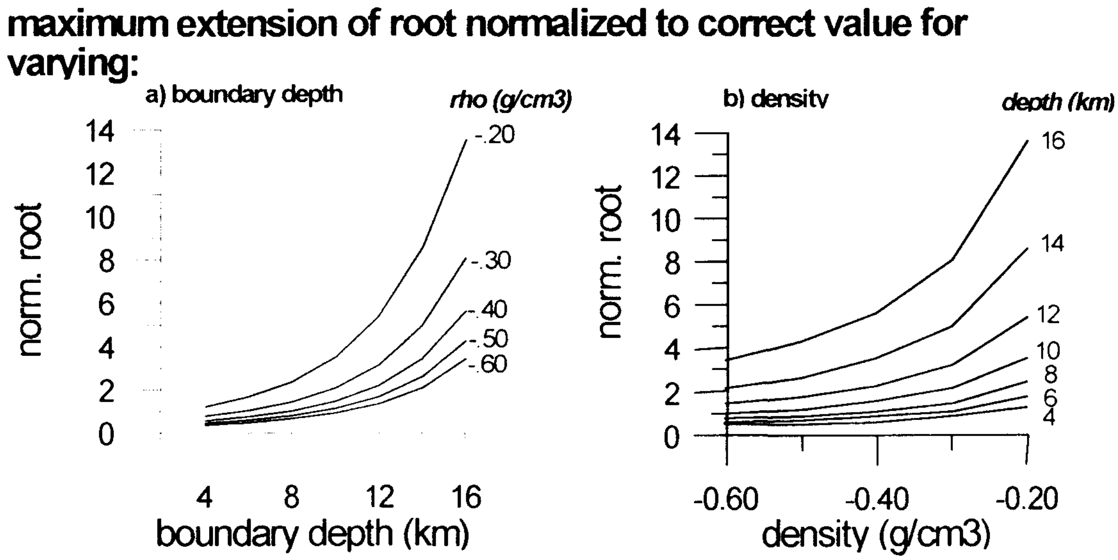

Fig. 3 - Maximum extension of root normalized to correct value. a) for varying reference depth at fixed density contrasts b) for varying density contrast at fixed reference depths.

The residual field is downward continued and a correction to the surface density of the sheet mass is obtained as in Eq. (2). The correction affects the oscillation amplitude of the density boundary according to Eq. (3). This procedure is repeated iteratively, obtaining at each iteration step k the residual gravity field $\delta g_k\ (x, y)$ and the oscillation amplitude of the boundary $r_k\ (x, y)$. The iterations are repeated until the root mean square (rms) gravity residual has reached an acceptable value or until the successive iteration does not give a considerable improvement to the results.

If seismic data regarding the depth of the studied boundary are available, they can be integrated into the inversion procedure. If the seismological depth estimation is available in definite points or along a profile, a set of inversions will be carried out for the different values of the reference depth (d) and the density contrast ($\Delta\rho$). The value (d) is accepted, for which the mean square difference between the depth of the boundary obtained from gravity inversion and from seismological investigations is minimal.

An important step in the inversion is the choice of the frequency limitation of the downward continuation, which is obtained with a frequency filter (h (γ)). The need for limitation is connected to the exponential multiplication factor that defines the continuation, and which makes the downward continuation unstable towards high frequencies. Besides the arguments of numerical kind, a problem inherent to the geophysical situation makes a limitation in frequency necessary. As the gravity field ($g_0\ (x, y)$) is explained by the oscillation of the boundary, the field ($g_0\ (x, y)$) should not contain energies due to masses posed above the oscillating boundary. Generally, the frequency range of the field generated by more superficial masses shifts to higher frequencies; therefore, the method of low-pass filtering the gravity data in the process of inversion manages, to a large extent, to separate the fields of masses posed above the boundary,

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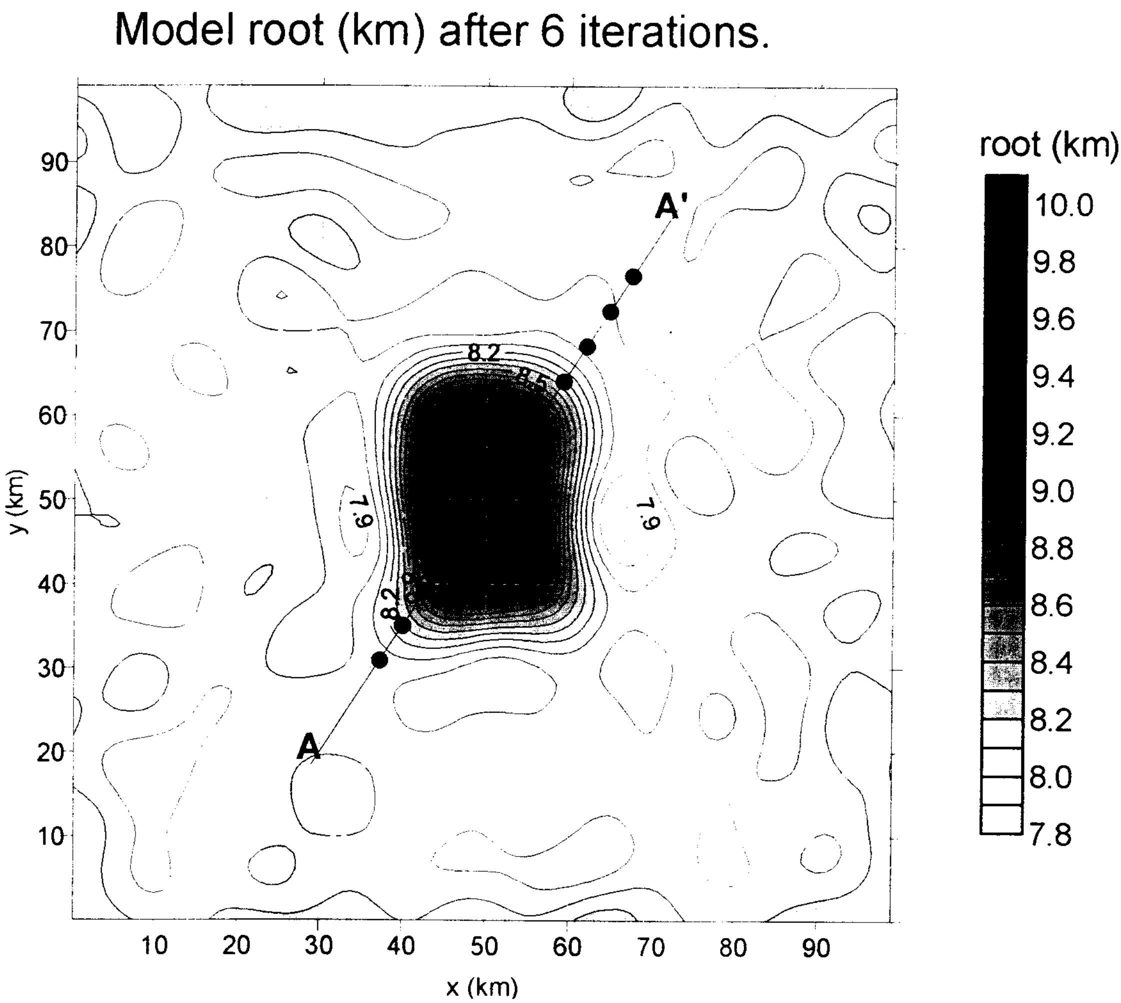

Fig. 4 - Map of the 3D inverted root after 6 iterations with position of constraining data (dots) and profile AA'.

if the filter is chosen appropriately.

2. Synthetic example

We test the inversion method on a synthetic Moho root. The root has the shape of two rectangular prisms set one on top of each other. Let the width, length and height of the upper prism be equal to 20 km, 30 km and 1km, that of the lower prism equal to 10 km, 10 km and 1 km, respectively. The top of the prisms is located at 8 km and 9 km, respectively. The lowest extension of the root is then at a 10 km depth and the density contrast is –0.4 g/cm^3. The prisms are centered on a 100×100 point grid of 1 km sampling. We add gaussian noise (standard deviation 0.5 mgal) to the synthetic gravity field. The gravity field is graphed in Fig. 1, the maximum anomaly having a value of –10 mGal. The black dots mark the positions where the

Table 1 - Root mean square deviation of the depths of the boundary obtained by the gravity inversion from the constraining depths. The root mean square deviation in km, for different density contrasts (g/cm³) and reference depths of the boundary (km).

	Reference depth of boundary (km)						
Density contrast (g/cm³)	*4*	*6*	*8*	*10*	*12*	*14*	*16*
-0.2	3.9	1.7	**1.3**	3.9	7.2	11.4	16.9
-0.3	4.3	2.1	**0.5**	2.7	5.3	8.5	12.4
-0.4	**4.4**	**2.3**	**0.3**	**2.2**	**4.6**	**7.4**	**10.6**
-0.5	4.5	2.4	**0.4**	1.9	4.2	6.8	9.7
-0.6	4.6	2.5	**0.5**	1.8	4.0	6.4	9.1

depth of the root will be acquired from secondary information sources, such as seismic sounding. First, a range of geophysically plausible values for the reference depth and density contrast of the boundary must be defined. In this case we allow the reference depth to vary between 4 km and 16 km and the density contrast between -0.2 g/cm^3 and -0.6 g/cm^3.

With P_m the cutoff wavelength, we define the filter function h (γ) as:

$$h(\gamma) = .5\left[1 + \cos(\pi\gamma P_m)\right]. \tag{5}$$

We choose the cutoff wavelength P_m= 11 km; the base of the prisms used for the direct calculation of the gravity field is square, with sides equal to 5 km.

The constraining values for the root are used to find the adequate density contrast and boundary reference depth. A certain amount of iterations is fixed (in this case 6) and for every couple a test inversion is made. In Table 1 the root mean square (rms) deviation of the inverted root from the constraining points is given for different couples of density and reference depths. The particular number of iterations chosen does not influence the resulting depth/density couple. It is seen that the couple with reference depth 8 km and density contrast -0.4 g/cm^3 is the one for which the rms deviation is minimal. The improvement of the model root and the modeled gravity field at every iteration step along profile AA' is shown in Fig. 2. After 6 iterative steps the iteration does not contribute to an appreciable enhancement of the gravity residual, and the iteration is ceased.

How the solution depends on the value of the boundary depth is shown in Fig. 3a, where for different density contrast (-0.2 g/cm^3 through -0.6 g/cm^3) the boundary depth varies in the range from 4 km to 16 km. The maximum penetration of the modeled boundary normalized by the synthetic model value (2 km), is graphed. For greater boundary depths the root is overestimated. A similar experiment is shown in Fig. 3b for different boundary, depths (4 km through 16 km and varying density contrast. The effect of the density contrasts is less, a small density contrast again leading to an overestimated root. The solution for the root is graphed in Fig. 4. The maximum depth of the modeled root is 10 km, which is the correct value.

3. Conclusion and discussion

We have discussed a 3D gravity inversion method which aims at modeling gravity data with the oscillations of an interface which marks a strong density contrast. The solution depends on the values of the density contrast and the reference depth of the boundary. These two parameters can be constrained if additional information from seismology defining the boundary depth (e.g. along a profile), is available. The method was applied and tested successfully for 2.5D models in the SE-Alps (Braitenberg et al., 1997) and in the Karakoram-Kohistan area (Braitenberg and Drigo, 1998). Subsequently, a 3D Moho inversion was applied to the SE-Alps on a geographic window of Longitude 10.5°-14.5°E and Latitude 45°-48°N (Zadro and Braitenberg, 1997). For details and results we refer to the cited papers.

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